

IB Mathematics A&A SL Review, Number and Algebra

This worksheet is due on Wed., Dec. 13 (7th), or Thurs., Dec. 14 (4th). **Work all problems on your own paper in black or dark blue ink.** Wherever possible, justify your answers by showing work. Numerical answers must be given exactly or to three significant figures unless otherwise specified in the problem. The assignment counts 30 points.

1. **(no calculator)** The values in the fourth row of Pascal's triangle are shown in the following table.

1	4	6	4	1
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- (a) Write down the values in the fifth row of Pascal's triangle.
(b) Hence or otherwise, find the term in x^3 in the expansion of $(2x + 3)^5$.
2. **(no calculator)** The first two terms of an infinite geometric sequence, in order, are $2 \log_2 x$, $\log_2 x$, where $x > 0$.
- (a) Find r .
(b) Show that the sum of the infinite sequence is $4 \log_2 x$.

The first three terms of an arithmetic sequence, in order, are $\log_2 x$, $\log_2 \left(\frac{x}{2}\right)$, $\log_2 \left(\frac{x}{4}\right)$, where $x > 0$.

- (c) Find d , giving your answer as an integer.
(d) Let S_{12} be the sum of the first 12 terms of the arithmetic sequence. Show that $S_{12} = 12 \log_2 x - 66$.
(e) Given that S_{12} is equal to half the sum of the infinite geometric sequence, find x , giving your answer in the form 2^p , where $p \in \mathbb{Q}$.
3. **(no calculator)** Consider the following sequence of figures.

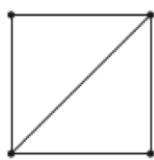


Figure 1

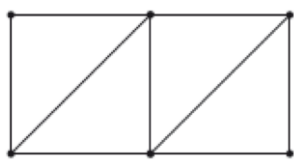


Figure 2

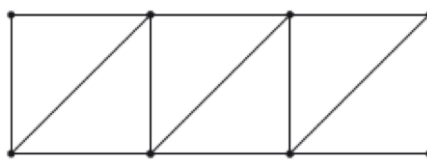


Figure 3

Figure 1 contains 5 line segments.

- (a) Given that Figure n contains 801 line segments, show that $n = 200$.
(b) Find the total number of line segments in the first 200 figures.

4. **(no calculator)** Let $x = \ln 3$ and $y = \ln 5$. Write the following expressions in terms of x and y .
- (a) $\ln\left(\frac{5}{3}\right)$
- (b) $\ln 45$
5. **(no calculator)** Three consecutive terms of a geometric sequence are $x - 3$, 6 , and $x + 2$. Find the possible values of x .
6. **(calculator allowed, but no CAS features)**
- (a) Find the term in x^6 in the expansion of $(x + 2)^9$.
- (b) Hence, find the term in x^7 in the expansion of $5x(x + 2)^9$.
7. **(calculator allowed, but no CAS features)** Consider the expansion of $\left(x^2 + \frac{2}{x}\right)^{10}$.
- (a) Write down the number of terms of this expansion.
- (b) Find the coefficient of x^8 .
8. **(no calculator)**
- (a) Given that $2^m = 8$ and $2^n = 16$, write down the value of m and of n .
- (b) Hence or otherwise solve $8^{2x+1} = 16^{2x-3}$.
9. **(no calculator)** The first three terms of a geometric sequence are $\ln x^{16}$, $\ln x^8$, $\ln x^4$, for $x > 0$.
- (a) Find the common ratio.
- (b) Solve $\sum_{k=1}^{\infty} 2^{5-k} \ln x = 64$.