

Today is about using regression to solve equations.

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PSAT

When should one use regression versus other methods?

If you want to solve a single-variable equation and it looks even a little bit annoying, I would recommend regression as a quick solution.

If you are asked to do what seems to you like a lot of algebraic manipulation in order to find the value of a parameter or a combination of parameters, regression may make that more doable. "What seems to you like a lot" is going to vary by person.

Use of regression ($\sim$) to solve equations, basic

Question 10298d3e

Show Algebra - Linear equations in one variable

If $2(3t - 10) + t = 40 + 4t$, what is the value of $3t$?

Answer

The correct answer is 60. Subtracting t from both sides of the given equation yields $2(3t - 10) = 40 + 3t$. Applying the distributive property to the left-hand side of this equation yields $6t - 20 = 40 + 3t$. Adding 20 to both sides of this equation yields $6t = 60 + 3t$. Subtracting $3t$ from both sides of this equation yields $3t = 60$. Therefore, the value of $3t$ is 60.

$$2(3t_1 - 10) + t_1 \sim 40 + 4t_1$$

REGRESSION PARAMETERS

$$t_1 = 20$$

STATISTICS
RMSE = 0

RESIDUALS
 e_1

$3t_1$

= 60

Question 1730960c

Advanced Math - Nonlinear equations in one variable and systems of equations in two variables

▼ 27/301

Show

$$\frac{2(x+1)}{x+5} = 1 - \frac{1}{x+5}$$

What is the solution to the equation above?

0

○ 2

○ 3

○ 5

$$\frac{2(x_1 + 1)}{x_1 + 5} \sim 1 - \frac{1}{x_1 + 5}$$

REGRESSION PARAMETERS

$x_1 = 2$

STATISTICS
 $RMSE =$

RESIDUALS
 e_1

Choice B is correct. Since $\frac{x+5}{x+5}$ is

equivalent to 1, the right-hand side of the given equation can be rewritten as

$\frac{x+5}{x+5} - \frac{1}{x+5}$, or $\frac{x+4}{x+5}$. Since the left- and

right-hand sides of the equation

$$\frac{2(x+1)}{x+5} = \frac{x+4}{x+5}$$
 have the same

denominator, it follows that $2(x+1) = x+4$.

Applying the distributive property of multiplication to the expression $2(x + 1)$

yields $2(x)+2(1)$, or $2x+2$. Therefore,

$2x + 2 = x + 4$. Subtracting x and 2 from both sides of this equation yields $x = 2$.

Regressions to find multiple solutions? Or should you graph?

Question 1a20a239

Advanced Math - Nonlinear equations in one variable and systems of equations in two variables

29/301

ABC

Show

$x^2 - 5x - 24 = 0$

What is the sum of the solutions to the given equation?

Answer

$a^2 - 5a - 24 \sim 0$

REGRESSION PARAMETERS
 $a = -3$

STATISTICS
 $RMSE = 0$

RESIDUALS
 e_1

$b^2 - 5b - 24 \sim 0 \{ b > -2.5 \}$

REGRESSION PARAMETERS
 $b = 8$

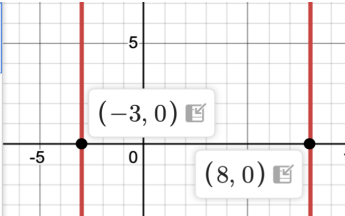
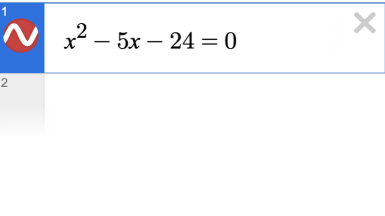
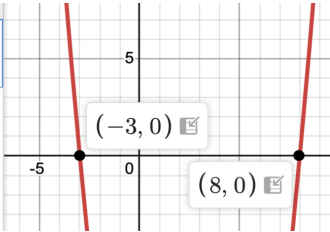
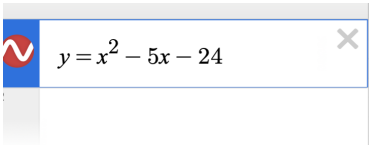
STATISTICS
 $RMSE = 0$

RESIDUALS
 e_2

$a + b$

= 5

The correct answer is 5. The given quadratic equation can be rewritten in factored form as $(x - 8)(x + 3) = 0$. Based on the zero product property, it follows that $x - 8 = 0$ or $x + 3 = 0$. Adding 8 to both sides of the equation $x - 8 = 0$ yields $x = 8$. Subtracting 3 from both sides of the equation $x + 3 = 0$ yields $x = -3$. Therefore, the solutions to the given equation are 8 and -3 . It follows that the sum of the solutions to the given equation is $8 + (-3)$, or 5.



...but also $-b/a$?

Use of ~ where it's necessary to give some values for x_1

(And how can you tell when that's likely to be needed?)

Question 91153f

Show Advanced Math - Equivalent expressions

should work for every x

In the expression $3(2x^2 + px + 8) - 16x(p + 4)$, p is a constant. This expression is equivalent to the expression $6x^2 - 155x + 24$. What is the value of p ?

- ☐ -3
- ☐ 7
- ☐ 13
- ☐ 155

×

Choice B is correct. Using the distributive property, the first given expression can be written as $6x^2 + 3px + 24 - 16px - 64x + 24$, and then rewritten as $6x^2 + (3p - 16p - 64)x + 48$. Since the expression $6x^2 + (3p - 16p - 64)x + 48$ is equivalent to $6x^2 - 155x + 24$, the coefficients of the x terms from each expression are equivalent to each other; thus $3p - 16p - 64 = -155$. Combining like terms gives $-13p - 64 = -155$. Adding 64 to both sides of the equation gives $-13p = -91$. Dividing both sides of the equation by -13 yields $p = 7$.

×

Choice A is incorrect. If $p = -3$, then the first expression would be equivalent to $6x^2 - 25x + 24$. Choice C is incorrect. If $p = 13$, then the first expression would be equivalent to $6x^2 - 233x + 24$. Choice D is incorrect. If $p = 155$, then the first expression would be equivalent to $6x^2 - 2,079x + 24$.

$3(2x_1^2 + px_1 + 8) - 16x_1(p + 4) \sim 6x_1^2 - 155x_1 + 24$

REGRESSION PARAMETERS
 $p = 7$
 STATISTICS RESIDUALS
 $RMSE = 6.55 \times 10^{-15} e_1$

$x_1 = \text{random}(5)$

= 0.706133194171 0.0389543268892 0.1730 [5 element list](#)

Question e732b9c

Show Advanced Math - Equivalent expressions

$$(ax + 3)(5x^2 - bx + 4) = 20x^3 - 9x^2 - 2x + 12$$

The equation above is true for all x , where a and b are constants. What is the value of ab ?

☐ 18

☐ 20

☐ 24

☐ 40

$(ax_1 + 3)(5x_1^2 - bx_1 + 4) \sim 20x_1^3 - 9x_1^2 - 2x_1 + 12$

REGRESSION PARAMETERS
 $a = 4$ $b = 6$
 STATISTICS RESIDUALS
 $RMSE = 1.12 \times 10^{-15} e_1$

$x_1 = \text{random}(5)$

= 0.238573485086 [5 element list](#)

ab

= 24

Choice C is correct. If the equation is true for all x , then the expressions on both sides of the equation will be equivalent. Multiplying the polynomials on the left-hand side of the equation gives $5ax^3 - abx^2 + 4ax + 15x^2 - 3bx + 12$. On the right-hand side of the equation, the only x^2 -term is $-9x^2$. Since the expressions on both sides of the equation are equivalent, it follows that $-abx^2 + 15x^2 = -9x^2$, which can be rewritten as $(-ab + 15)x^2 = -9x^2$. Therefore, $-ab + 15 = -9$, which gives $ab = 24$.

×

×

More use of ~ where it's necessary to give some values for the variable

Question 137cc6fd

Show Advanced Math - Equivalent expressions

$$\sqrt[5]{70n}(\sqrt[6]{70n})^2$$

For what value of x is the given expression equivalent to $(70n)^{30x}$, where $n > 1$?

Answer

1

$$\sqrt[5]{70n}(\sqrt[6]{70n})^2 \sim (70n)^{x_1}$$

REGRESSION PARAMETERS
 $x_1 = 0.0177778$

STATISTICS
 $RMSE = 1.91 \times 10^{-15}$

RESIDUALS
 e_1

2

$n = \text{random}(8)$
 $= 0.979625772636$ [8 element list](#)

3

x_1
 $= 0.0177777777778$

4

x_1
 $= \frac{4}{225}$

The correct answer is $\frac{4}{225}$. An expression of the form $\sqrt[k]{a}$, where k is an integer greater than 1 and $a \geq 0$, is equivalent to $a^{\frac{1}{k}}$. Therefore, the given expression, where $n > 1$, is equivalent to $(70n)^{\frac{1}{5}}((70n)^{\frac{1}{6}})^2$. Applying properties of exponents, this expression can be rewritten as $(70n)^{\frac{1}{5}}(70n)^{\frac{1}{3}}$, or $(70n)^{\frac{1}{5} + \frac{1}{3}}$, which can be rewritten as $(70n)^{\frac{8}{15}}$, or $(70n)^{\frac{8}{15}}$. It's given that the expression $\sqrt[5]{70n}(\sqrt[6]{70n})^2$ is equivalent to $(70n)^{30x}$, where $n > 1$. It follows that $(70n)^{\frac{8}{15}}$ is equivalent to $(70n)^{30x}$. Therefore, $\frac{8}{15} = 30x$. Dividing both sides of this equation by 30 yields $\frac{8}{450} = x$, or $\frac{4}{225} = x$. Thus, the value of x for which the given expression is equivalent to $(70n)^{30x}$, where $n > 1$, is $\frac{4}{225}$. Note that 4/225, .0177, .0178, 0.017, and 0.018 are examples of ways to enter a correct answer.

0.0178
0.0177
0.0178
0.0177

Question 2730fe3d

Advanced Math - Equivalent expressions

45/301

Show

$$(7532 + 100y^2) + 10(10y^2 - 110)$$

The expression above can be written in the form $ay^2 + b$, where a and b are constants. What is the value of $a + b$?

Answer

$$(7532 + 100y_1^2) + 10(10y_1^2 - 110) \sim ay_1^2 + b$$

REGRESSION PARAMETERS
 $a = 200$ $b = 6432$

STATISTICS
 $RMSE = 7.04 \times 10^{-13}$

RESIDUALS
 e_1

$a + b$
 $= 6632$

$y_1 = \text{random}(5)$
 $= 0.0207997785752$ 0.79909065201 0.927 [5 element list](#)

The correct answer is 6632. Applying the distributive property to the expression yields $(7532 + 100y^2) + (100y^2 - 1100)$. Then adding together $7532 + 100y^2$ and $100y^2 - 1100$ and collecting like terms results in $200y^2 + 6432$. This is written in the form $ay^2 + b$, where $a = 200$ and $b = 6432$. Therefore $a + b = 200 + 6432 = 6632$.

Use of ~ where only a relationship between parameters is needed

Question 12e7faf8

Advanced Math - Equivalent expressions

23/301

Show

The equation $\frac{x^2+6x-7}{x+7} = ax+d$ is true for all $x \neq -7$, where

a and d are integers. What is the value of $a+d$?

☐ -6

☐ -1

☐ 0

☐ 1

$$\frac{x_1^2+6x_1-7}{x_1+7} \sim ax_1+d$$

REGRESSION PARAMETERS

$x_1=1$

$a=-1$

$d=1$

STATISTICS

$RMSE=0$

RESIDUALS

e_1

$a+d$

= 0

Choice C is correct. Since the expression x^2+6x-7 can be factored as $(x+7)(x-1)$, the given equation can be rewritten as $\frac{(x+7)(x-1)}{x+7} = ax+d$. Since $x \neq -7$, $x+7$ is also not equal to 0, so both the numerator and denominator of $\frac{(x+7)(x-1)}{x+7}$ can be divided by $x+7$. This gives $x-1 = ax+d$. Equating the coefficient of x on each side of the equation gives $a=1$. Equating the constant terms gives $d=-1$. The sum is $1+(-1)=0$.

x

Question 34847f8a

Advanced Math - Equivalent expressions

58/301

Show

$$\frac{2}{x-2} + \frac{3}{x+5} = \frac{rx+t}{(x-2)(x+5)}$$

The equation above is true for all $x > 2$, where r and t are positive constants. What is the value of rt ?

☐ -20

☐ 15

☐ 20

☐ 60

$$\frac{2}{x_1-2} + \frac{3}{x_1+5} \sim \frac{rx_1+t}{(x_1-2)(x_1+5)}$$

REGRESSION PARAMETERS

$r=5$

$t=4$

STATISTICS

$RMSE=0$

RESIDUALS

e_1

rt

= 20

$x_1 = \text{random}(5)$

= 0.326699456298 0.51402725301 0.9061 [5 element list](#)

Choice C is correct. To express the sum of the two rational expressions on the left-hand side of the equation as the single rational expression on the right-hand side of the equation, the expressions on the left-hand side must have the same denominator. Multiplying the first expression by $\frac{x+5}{x-5}$ results in $\frac{2(x+5)}{(x-2)(x+5)}$, and multiplying the second expression by $\frac{x-2}{x-2}$ results in $\frac{3(x-2)}{(x-2)(x+5)}$, so the given equation can be rewritten as $\frac{2(x+5)}{(x-2)(x+5)} + \frac{3(x-2)}{(x-2)(x+5)} = \frac{rx+t}{(x-2)(x+5)}$, or $\frac{2x+10}{(x-2)(x+5)} + \frac{3x-6}{(x-2)(x+5)} = \frac{rx+t}{(x-2)(x+5)}$. Since the two rational expressions on the left-hand side of the equation have the same denominator as the rational expression on the right-hand side of the equation, it follows that $(2x+10)+(3x-6)=rx+t$. Combining like terms on the left-hand side yields $5x+4=rx+t$, so it follows that $r=5$ and $t=4$. Therefore, the value of rt is $(5)(4)=20$.

More use of ~ where only a relationship between parameters is needed

Question 40c09d66

Show Advanced Math - Equivalent expressions

If $\frac{\sqrt{x^5}}{\sqrt[3]{x^4}} = x^{\frac{a}{b}}$ for all positive values of x, what is the value of $\frac{a}{b}$?

Answer

1

|

$$\frac{\sqrt{x_1^5}}{\sqrt[3]{x_1^4}} \sim x_1^{\frac{a}{b}}$$

REGRESSION PARAMETERS

$a = 1.07698$

$b = 0.923128$

STATISTICS

$RMSE = 0$

RESIDUALS

e_1

2

$\frac{a}{b}$

=

1.16666666667

3

$x_1 = \text{random}(5)$

=

0.952817832393

0.5585

5 element list

4

$\frac{a}{b}$

=

$\frac{7}{6}$

The correct answer is $\frac{7}{6}$. The value of $\frac{a}{b}$ can be found by first rewriting the left-hand side of the given equation as $x^{\frac{\frac{5}{2}}{\frac{4}{3}}}$. Using the properties of exponents, this expression can be rewritten as $x^{\left(\frac{5}{2} - \frac{4}{3}\right)}$. This expression can be rewritten by subtracting the fractions in the exponent, which yields $x^{\frac{7}{6}}$. Thus, $\frac{a}{b}$ is $\frac{7}{6}$. Note that 7/6, 1.166, and 1.167 are examples of ways to enter a correct answer.

Regression with systems of equations

Question 45538ac6



Algebra - Systems of two linear equations in two variables

87/301



Show

$$-x - wy = -337$$

$$2x - wy = 47$$

$$[-x - wy, 2x - wy] \sim [-337, 47]$$

In the given system of equations, w is a constant. In the xy -plane, the graphs of these equations intersect at the point $(q, 19)$, where q is a constant. What is the value of w ?

Answer

$$[-q - w \cdot 19, 2q - w \cdot 19] \sim [-337, 47]$$

REGRESSION PARAMETERS

 $q = 128$ $w = 11$

STATISTICS

RESIDUALS

RMSE = 0

 e_1

The correct answer is 11. It's given that the graphs of the equations in the given system intersect at the point $(q, 19)$, where q is a constant. Therefore, the coordinates of this point must satisfy both equations. Substituting the point $(q, 19)$ into the first equation, $-x - wy = -337$, yields $-q - w(19) = -337$. Adding $19w$ to both sides of this equation yields $-q = -337 + 19w$, which is equivalent to $q = 337 - 19w$. Substituting the point $(q, 19)$ into the second equation yields $2q - w(19) = 47$. Substituting $337 - 19w$ in place of q in the equation $2q - w(19) = 47$ yields $2(337 - 19w) - 19w = 47$. Applying the distributive property to the left-hand side of this equation yields $674 - 38w - 19w = 47$. Combining like terms on the left-hand side of this equation yields $674 - 57w = 47$. Subtracting 674 from both sides of this equation yields $-57w = -627$. Dividing both sides of this equation by -57 yields $w = 11$.

Question 92ae7544



Show Advanced Math - Nonlinear functions

$$f(0) - 12 = -\frac{75}{7}$$

$$ab = \frac{320}{7}$$

Function f is defined by $f(x) = -a^x + b$, where a and b are constants. In the xy -plane, the graph of $y = f(x) - 12$ has a y -intercept at $(0, -\frac{75}{7})$. The product of a and b is $\frac{320}{7}$. What is the value of a ?

Answer

$$f(x) = -a^x + b$$

$$[f(0) - 12, ab] \sim [-\frac{75}{7}, \frac{320}{7}]$$

REGRESSION PARAMETERS

 $a = 20$ $b = 2.28571$

STATISTICS

RESIDUALS

RMSE = 0

 e_1

The correct answer is 20. It's given that $f(x) = -a^x + b$. Substituting $-a^x + b$ for $f(x)$ in the equation $y = f(x) - 12$ yields $y = -a^x + b - 12$. It's given that the y -intercept of the graph of $y = f(x) - 12$ is $(0, -\frac{75}{7})$. Substituting 0 for x and $-\frac{75}{7}$ for y in the equation $y = -a^x + b - 12$ yields $-\frac{75}{7} = -a^0 + b - 12$, which is equivalent to $-\frac{75}{7} = -1 + b - 12$, or $-\frac{75}{7} = b - 13$. Adding 13 to both sides of this equation yields $\frac{16}{7} = b$. It's given that the product of a and b is $\frac{320}{7}$, or $ab = \frac{320}{7}$. Substituting $\frac{16}{7}$ for b in this equation yields $a(\frac{16}{7}) = \frac{320}{7}$. Dividing both sides of this equation by $\frac{16}{7}$ yields $a = 20$.

More regression with systems of equations

Question c56905b0

Algebra - Linear equations in two variables

▼ 230/301

Show

The graph of the equation $ax + ky = 6$ is a line in the xy -plane, where a and k are constants. If the line contains the points $(-2, -6)$ and $(0, -3)$, what is the value of k ?

- ☐ -2
- ☐ -1
- ☐ 2
- ☐ 3

Choice A is correct. The value of k can be found using the slope-intercept form of a linear equation, $y = mx + b$, where m is the slope and b is the y -coordinate of the y -intercept. The equation $ax + ky = 6$ can be rewritten in the form $y = -\frac{ax}{k} + \frac{6}{k}$. One of the given points, $(0, -3)$, is the y -intercept. Thus, the y -coordinate of the y -intercept -3 must be equal to $\frac{6}{k}$. Multiplying both sides by k gives $-3k = 6$. Dividing both sides by -3 gives $k = -2$.

$[a(-2) + k(-6), a \cdot 0 + k \cdot -3] \sim [6, 6]$

REGRESSION PARAMETERS

$a = 3$

$k = -2$

STATISTICS

$RMSE = 0$

RESIDUALS

e_1

Use of ~ where it helps to define a function first

Question b76bb67

Show Advanced Math - Nonlinear functions

Let the function p be defined as $p(x) = \frac{(x - c)^2 + 160}{2c}$,

where c is a constant. If $p(c) = 10$, what is the value of $p(12)$?

☐ 10.00

☐ 10.25

☐ 10.75

☐ 11.00

$p(x) = \frac{(x - c)^2 + 160}{2c}$

$p(c) \sim 10$

REGRESSION PARAMETERS
 $c = 8$

STATISTICS
 $RMSE = 0$

RESIDUALS
 e_1

$p(12)$

= 11

Choice D is correct. The value of $p(12)$ depends on the value of the constant c , so the value of c must first be determined. It is given that $p(c) = 10$. Based on the definition of p , it follows that:

$$p(c) = \frac{(c - c)^2 + 160}{2c} = 10$$
$$\frac{160}{2c} = 10$$
$$2c = 16$$
$$c = 8$$

This means that $p(x) = \frac{(x - 8)^2 + 160}{16}$ for all values of x .

Therefore:

$$p(12) = \frac{(12 - 8)^2 + 160}{16}$$
$$= \frac{16 + 160}{16}$$
$$= 11$$

Question ea182446

Show Advanced Math - Nonlinear functions

$$h(x) = x^3 + ax^2 + bx + c$$

The function h is defined above, where a , b , and c are integer constants. If the zeros of the function are -5 , 6 , and 7 , what is the value of c ?

Answer

$h(x) = x^3 + ax^2 + bx + c$

$[h(-5), h(6), h(7)] \sim [0, 0, 0]$

REGRESSION PARAMETERS
 $a = -8$ $b = -23$ $c = 210$

STATISTICS
 $RMSE = 0$

RESIDUALS
 e_1

$$h(-5) = 0$$
$$h(6) = 0$$
$$h(7) = 0$$

The correct answer is 210. Since -5 , 6 , and 7 are zeros of the function, the function can be rewritten as $h(x) = (x + 5)(x - 6)(x - 7)$. Expanding the function yields $h(x) = x^3 - 8x^2 - 23x + 210$. Thus, $a = -8$, $b = -23$, and $c = 210$. Therefore, the value of c is 210.

Regression for factoring

Question 3cbafdee
Advanced Math - Equivalent expressions

75/301

Show

The expression $\frac{1}{3}x^2 - 2$ can be rewritten as $\frac{1}{3}(x - k)(x + k)$, where k is a positive constant. What is the value of k ?

- ☐ 2
- ☐ 6
- ☐ $\sqrt{2}$
- ☐ $\sqrt{6}$

Choice D is correct. Factoring out the coefficient $\frac{1}{3}$, the given expression can be rewritten as $\frac{1}{3}(x^2 - 6)$. The expression $x^2 - 6$ can be approached as a difference of squares and rewritten as $(x - \sqrt{6})(x + \sqrt{6})$. Therefore, k must be $\sqrt{6}$.

$$\frac{1}{3}x_1^2 - 2 \sim \frac{1}{3}(x_1 - k)(x_1 + k)$$

REGRESSION PARAMETERS

$x_1 = 1$

STATISTICS

$RMSE = 0$

$k = 2.44949$

RESIDUALS

e_1

$\sqrt{(6)}$

= 2.44948974278

Question 4d9b87ba
Show Advanced Math - Equivalent expressions

This one requires checking the choices with regression.

If $a = c + d$, which of the following is equivalent to the expression $x^2 - c^2 - 2cd - d^2$?

- ☐ $(x + a)^2$
- ☐ $(x - a)^2$
- ☐ $(x + a)(x - a)$
- ☐ $x^2 - ax - a^2$

Choice C is correct. Factoring -1 from the second, third, and fourth terms gives $x^2 - c^2 - 2cd - d^2 = x^2 - (c^2 + 2cd + d^2)$. The expression $c^2 + 2cd + d^2$ is the expanded form of a perfect square: $c^2 + 2cd + d^2 = (c + d)^2$. Therefore, $x^2 - (c^2 + 2cd + d^2) = x^2 - (c + d)^2$. Since $a = c + d$, $x^2 - (c + d)^2 = x^2 - a^2$. Finally, because $x^2 - a^2$ is the difference of squares, it can be expanded as $x^2 - a^2 = (x + a)(x - a)$.

$$x_1^2 - c^2 - 2cd - d^2 \sim (x_1 + a)^2$$

REGRESSION PARAMETERS

$x_1 = 0.580349$

$c = 0.112415$

$d = 0.112415$

$a = -0.0453195$

STATISTICS

RESIDUALS

$RMSE = 0$

e_1

$c + d$

= 0.224829847126

a

= -0.0453195254486

$$x_1^2 - c^2 - 2cd - d^2 \sim (x_1 + a)(x_1 - a)$$

REGRESSION PARAMETERS

$x_1 = 1$

$c = 0.608756$

$d = 0.608756$

$a = 1.21751$

STATISTICS

RESIDUALS

$RMSE = 0$

e_1

$c + d$

= 1.21751271026

a

= 1.21751271026

$$x_1^2 - c^2 - 2cd - d^2 \sim (x_1 - a)^2$$

REGRESSION PARAMETERS

$x_1 = 0.377373$

$c = 0.100797$

$d = 0.100797$

$a = 0.0583582$

STATISTICS

RESIDUALS

$RMSE = 0$

e_1

$c + d$

= 0.201593453086

a

= 0.0583582347816

$$x_1^2 - c^2 - 2cd - d^2 \sim x_1^2 - ax_1 - a^2$$

REGRESSION PARAMETERS

$x_1 = 1.09167$

$c = 0.798056$

$d = 0.798056$

$a = 1.15466$

STATISTICS

RESIDUALS

$RMSE = 0$

e_1

$c + d$

= 1.59611167247

a

= 1.15466434626