

Next Monday is Labor Day — no school, so we won't meet next week.

The following Monday, Sept. 14, is the day I'm called for jury duty.

As we discussed in the first session, I estimate a better than 50% chance that I will still be able to get here for class. However, I won't know for sure until much closer to that time.

If I find out I *do* have to miss class, I'll e-mail you as soon as I know that. It's possible I won't know until that Monday morning, though.

Aug. 31 (today): geometry and trig

Sept. 7: no class, Labor Day holiday

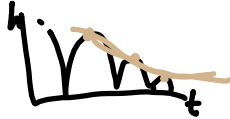
Sept. 14 (👉): Desmos tables and basic regression tools

Sept. 21: more advanced regression techniques in Desmos

Sept. 28: problem-solving and data analysis type questions

Oct. 5:

Math: Question 11



A ball is dropped from an initial height of 22 feet and bounces off the ground repeatedly. The function h estimates that the maximum height reached after each time the ball hits the ground is 85% of the maximum height reached after the previous time the ball hit the ground. Which equation defines h , where $h(n)$ is the estimated maximum height of the ball after it has hit the ground n times and n is a whole number greater than 1 and less than 10?

$n = 0$

- ☐ $h(n) = 22(0.22)^n$
- ☒ $h(n) = 22(0.85)^n$
- ☐ $h(n) = 85(0.22)^n$
- ☐ $h(n) = 85(0.85)^n$

Answer

You selected answer B. The correct answer is B.

Rationale

Choice B is correct. It's given that for the function h , $h(n)$ is the estimated maximum height, in feet, of the ball after it has hit the ground n times. It's also given that the function h estimates that the maximum height reached after each time the ball hits the ground is 85% of the maximum height reached after the previous time the ball hit the ground. It follows that h is a decreasing exponential function that can be written in the form $h(n) = a\left(\frac{p}{100}\right)^n$, where a is the initial height, in feet, the ball was dropped from and the function estimates that the maximum height reached after each time the ball hits the ground is $p\%$ of the maximum height reached after the previous time the ball hit the ground. It's given that the ball is dropped from an initial height of 22 feet. Therefore, $a = 22$. Since the function h estimates that the maximum height reached after each time the ball hits the ground is 85% of the maximum height reached after the previous time the ball hit the ground, $p = 85$. Substituting 22 for a and 85 for p in the equation $h(n) = a\left(\frac{p}{100}\right)^n$ yields $h(n) = 22\left(\frac{85}{100}\right)^n$, or $h(n) = 22(0.85)^n$.

5. p. 29 Ch. 5 #40

40. For the equation below, which of the following expresses x in terms of y , and z ? 4

$$y = \frac{xz - z^2}{x - 1}$$

$x = \text{stuff w/ } y \text{ and } z$

in it

Therefore, solve for x

A) $\frac{z^2 - y}{z - y}$

B) $\frac{1 + y}{z^2 + y}$

C) $\frac{y - z^2}{z + 1}$

D) $\frac{z^2 + y}{1 - y}$

$$(x - 1)y = \frac{xz - z^2}{x - 1} \cdot (x - 1)$$

$$(x - 1)y = xz - z^2$$

$$\begin{array}{r} xy - y \\ -xz + y \\ \hline \end{array} = \begin{array}{r} xz - z^2 \\ -xz + y \\ \hline \end{array}$$

$$xy - xz = y - z^2$$

$$\frac{x(y - z)}{y - z} = \frac{y - z^2}{y - z}$$

$$x = \frac{y - z^2}{y - z} \cdot \frac{-1}{-1} = \frac{z^2 - y}{z - y}$$

Geometry and trigonometry is broken down into four subsections:

- Area and volume
- Lines, angles, and triangles
- Right triangles and trigonometry
- Circles

• Area and volume

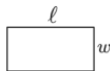
Area and volume formulas are provided in Bluebook. You should know what's there and look at those formulas if you are unsure of one.

Reference



$$A = \pi r^2$$

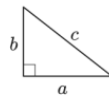
$$C = 2\pi r$$



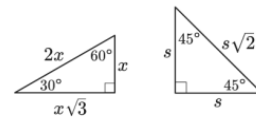
$$A = \ell w$$



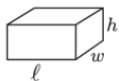
$$A = \frac{1}{2}bh$$



$$c^2 = a^2 + b^2$$



Special Right Triangles



$$V = \ell wh$$



$$V = \pi r^2 h$$



$$V = \frac{4}{3}\pi r^3$$



$$V = \frac{1}{3}\pi r^2 h$$



$$V = \frac{1}{3}\ell wh$$

The number of degrees of arc in a circle is 360.

The number of radians of arc in a circle is 2π .

The sum of the measures in degrees of the angles of a triangle is 180.

Question 2cc6e02

Show Geometry and Trigonometry - Area and volume

The volume of right circular cylinder A is 22 cubic centimeters. What is the volume, in cubic centimeters, of a right circular cylinder with twice the radius and half the height of cylinder A?

☐ 11

☐ 22

☒ 44

☐ 66

Choice C is correct. The volume of right circular cylinder A is given by the expression $\pi r^2 h$, where r is the radius of its circular base and h is its height. The volume of a cylinder with twice the radius and half the height of cylinder A is given by $\pi(2r)^2\left(\frac{1}{2}h\right)$, which is equivalent to $4\pi r^2\left(\frac{1}{2}h\right) = 2\pi r^2 h$. Therefore, the volume is twice the volume of cylinder A, or $2 \times 22 = 44$.

Choice A is incorrect and likely results from not multiplying the radius of cylinder A by 2. Choice B is incorrect and likely results from not squaring the 2 in $2r$ when applying the volume formula. Choice D is incorrect and likely results from a conceptual error.

Question a44345a

Geometry and Trigonometry - Area and volume

☐ 2/39


Show

Parallelogram $ABCD$ is similar to parallelogram $PQRS$. The length of each side of parallelogram $PQRS$ is 2 times the length of its corresponding side of parallelogram $ABCD$. The area of parallelogram $ABCD$ is 5 square centimeters. What is the area, in square centimeters, of parallelogram $PQRS$?

☐ 7

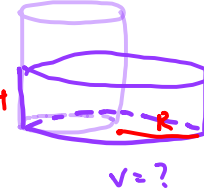
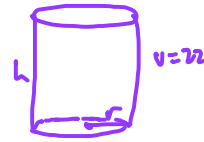
☐ 10

☐ 20

☐ 25

Choice C is correct. It's given that parallelogram $ABCD$ is similar to parallelogram $PQRS$. When two parallelograms are similar, if the scale factor between their corresponding side lengths is k , the scale factor between their areas is k^2 . It's given that the length of each side of parallelogram $PQRS$ is 2 times the length of its corresponding side of parallelogram $ABCD$, so the scale factor between their corresponding side lengths is 2. It follows that the scale factor between their areas is 2^2 , or 4. It's given that the area, in square centimeters, of parallelogram $ABCD$ is 5. It follows that the area, in square centimeters, of parallelogram $PQRS$ is $5(4)$, or 20.

$$V = \pi r^2 h = 22$$



$$\begin{aligned} V &= \pi \cdot (R)^2 \cdot (H) \\ &= \pi \cdot (2r)^2 \cdot \left(\frac{1}{2}h\right) \\ &= \pi \cdot 4r^2 \cdot \frac{1}{2}h = 2\pi r^2 h \\ &= 22 \cdot 2 = 44 \end{aligned}$$

Question c8ef2ee

Geometry and Trigonometry - Area and volume

3/39



Show

A cube has a surface area of 54 square meters. What is the volume, in cubic meters, of the cube?


☐ 18

☐ 27

☐ 36

☐ 81

Choice B is correct. The surface area of a cube with side length s is equal to $6s^2$. Since the surface area is given as 54 square meters, the equation $54 = 6s^2$ can be used to solve for s . Dividing both sides of the equation by 6 yields $9 = s^2$. Taking the square root of both sides of this equation yields $3 = s$ and $-3 = s$. Since the side length of a cube must be a positive value, $s = -3$ can be discarded as a possible solution, leaving $s = 3$. The volume of a cube with side length s is equal to s^3 . Therefore, the volume of this cube, in cubic meters, is 3^3 , or 27.

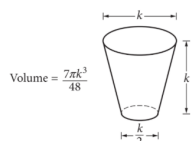
Question 1d2c5e42

Geometry and Trigonometry - Area and volume

5/39



Show



$$\frac{7\pi k^3}{48} = 16$$

The glass pictured above can hold a maximum volume of 473 cubic centimeters, which is approximately 16 fluid ounces. What is the value of k , in centimeters?

☐ 2.52

☐ 7.67

☐ 7.79

☐ 10.11

Choice D is correct. Using the volume formula

$V = \frac{7\pi k^3}{48}$ and the given information that the volume of the glass is 473 cubic centimeters, the value of k can be found as follows:

$$473 = \frac{7\pi k^3}{48}$$

$$k^3 = \frac{473(48)}{7\pi}$$

$$k = \sqrt[3]{\frac{473(48)}{7\pi}} \approx 10.10690$$

Therefore, the value of k is approximately 10.11 centimeters.

Choices A, B, and C are incorrect. Substituting the values of k from these choices in the formula results in volumes of approximately 7 cubic centimeters, 207 cubic centimeters, and 217 cubic centimeters, respectively, all of which contradict the given information that the volume of the glass is 473 cubic centimeters.

Question 20457c16

Geometry and Trigonometry - Area and volume

7/39



Show

A circle has a circumference of 31π centimeters. What is the diameter, in centimeters, of the circle?

Answer



The correct answer is 31. The circumference of a circle is equal to $2\pi r$ centimeters, where r represents the radius, in centimeters, of the circle, and the diameter of the circle is equal to $2r$ centimeters. It's given that a circle has a circumference of 31π centimeters. Therefore, $31\pi = 2\pi r$. Dividing both sides of this equation by π yields $31 = 2r$. Since the diameter of the circle is equal to $2r$ centimeters, it follows that the diameter, in centimeters, of the circle is 31.

Question 6c57095a

Geometry and Trigonometry - Area and volume

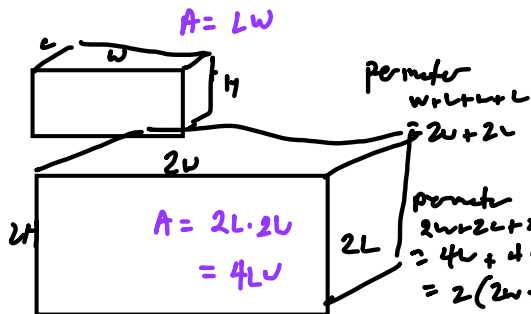
14/39



Show

A rectangular poster has an area of 360 square inches. A copy of the poster is made in which the length and width of the original poster are each increased by 20%. What is the area of the copy, in square inches?

Answer



The correct answer is 518.4. It's given that the area of the original poster is 360 square inches. Let ℓ represent the length, in inches, of the original poster, and let w represent the width, in inches, of the original poster. Since the area of a rectangle is equal to its length times its width, it follows that $360 = \ell w$. It's also given that a copy of the poster is made in which the length and width of the original poster are each increased by 20%. It follows that the length of the copy is the length of the original poster plus 20% of the length of the original poster, which is equivalent to $\ell + \frac{20}{100}\ell$ inches. This length can be rewritten as $\ell + 0.2\ell$ inches, or 1.2ℓ inches. Similarly, the width of the copy is the width of the original poster plus 20% of the width of the original poster, which is equivalent to $w + \frac{20}{100}w$ inches. This width can be rewritten as $w + 0.2w$ inches, or $1.2w$ inches. Since the area of a rectangle is equal to its length times its width, it follows that the area, in square inches, of the copy is equal to $(1.2\ell)(1.2w)$, which can be rewritten as $(1.2)(1.2)(\ell w)$. Since $360 = \ell w$, the area, in square inches, of the copy can be found by substituting 360 for ℓw in the expression $(1.2)(1.2)(\ell w)$, which yields $(1.2)(1.2)(360)$, or 518.4. Therefore, the area of the copy, in square inches, is 518.4.

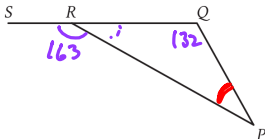
$$\begin{aligned} \text{New area} \\ 1.2L \cdot 1.2w \\ = 1.44Lw \end{aligned}$$

• Lines, angles, and triangles

Question 3fe928e2
Geometry and Trigonometry - Lines, angles, and triangles

7/34

Show



Note: Figure not drawn to scale.

In triangle PQR , $\overline{QR}$ is extended to point S . The measure of $\angle PQR$ is 132° , and the measure of $\angle PRS$ is 163° . What is the measure of $\angle QPR$?

- ☐ 48°
- ☐ 31°
- ☐ 24°
- ☐ 17°

Choice B is correct. In the figure shown, since $\overline{QS}$ is a line segment, the sum of the measures of $\angle PRS$ and $\angle PRQ$ is 180° . It's given that the measure of $\angle PRS$ is 163° . Thus, the measure of $\angle PRQ$ is $(180 - 163)^\circ$, or 17° . The sum of the measures of the interior angles of a triangle is 180° . It's given that the measure of $\angle PQR$ is 132° . Therefore, the measure of $\angle QPR$ is $(180 - 17 - 132)^\circ$, or 31° .

Choice A is incorrect. This is the measure of the supplement of $\angle PQR$, not the measure of $\angle QPR$.

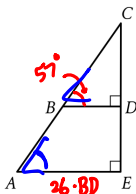
Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect. This is the measure of $\angle PRQ$, not the measure of $\angle QPR$.

Question 27c00a2
Geometry and Trigonometry - Lines, angles, and triangles

1/34

Show



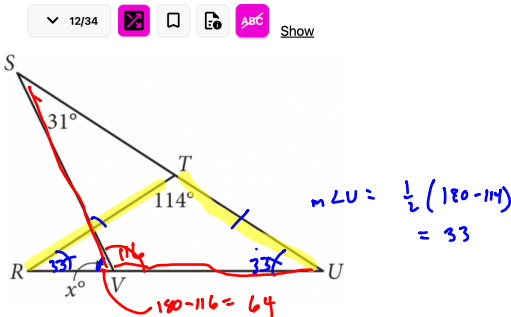
Note: Figure not drawn to scale.

In the figure shown, triangle CAE is similar to triangle CBD . The measure of angle CBD is 57° , and $AE = 26(BD)$. What is the measure of angle CAE ?

- ☐ $(26 \cdot 57)^\circ$
- ☐ $(26 + 57)^\circ$
- ☒ 57°
- ☐ 26°

Choice C is correct. It's given that triangle CAE is similar to triangle CBD . Corresponding angles in similar triangles have equal measure. Angle BCD and angle ACE represent the same angle. It follows that angle BCD and angle ACE have equal measure and are corresponding angles. It's given in the figure that angle BDC and angle AEC are right angles and therefore have equal measure. It follows that angle BDC and angle AEC are corresponding angles. Therefore, angle CBD and angle CAE are corresponding angles and have equal measure. It's given that the measure of angle CBD is 57° , so the measure of angle CAE is 57° .

Question 66ac9e77
Geometry and Trigonometry - Lines, angles, and triangles



In the figure above, $RT = TU$. What is the value of x ?

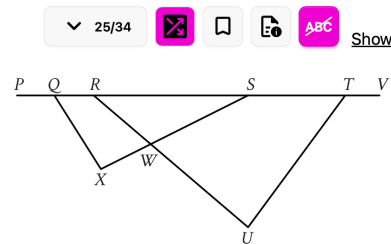
- ☐ 72
- ☐ 66
- ☐ 64
- ☐ 58

Choice C is correct. Since $RT = TU$, it follows that $\triangle RTU$ is an isosceles triangle with base RU . Therefore, $\angle TRU$ and $\angle TUR$ are the base angles of an isosceles triangle and are congruent. Let the measures of both $\angle TRU$ and $\angle TUR$ be t° . According to the triangle sum theorem, the sum of the measures of the three angles of a triangle is 180° . Therefore, $114^\circ + 2t^\circ = 180^\circ$, so $t = 33$.

Note that $\angle TUR$ is the same angle as $\angle SUV$. Thus, the measure of $\angle SUV$ is 33° . According to the triangle exterior angle theorem, an external angle of a triangle is equal to the sum of the opposite interior angles. Therefore, x° is equal to the sum of the measures of $\angle VSU$ and $\angle SUV$; that is, $31^\circ + 33^\circ = 64^\circ$. Thus, the value of x is 64.

Choice B is incorrect. This is the measure of $\angle STR$, but $\angle STR$ is not congruent to $\angle SVR$. Choices A and D are incorrect and may result from a calculation error.

Question b2426a45
Geometry and Trigonometry - Lines, angles, and triangles



Note: Figure not drawn to scale.

In the figure shown, points Q , R , S , and T lie on line segment PV , and line segment RU intersects line segment SX at point W . The measure of $\angle SQX$ is 48° , the measure of $\angle SXQ$ is 86° , the measure of $\angle SWU$ is 85° , and the measure of $\angle VTU$ is 162° . What is the measure, in degrees, of $\angle TUR$?

Answer

The correct answer is 123. The triangle angle sum theorem states that the sum of the measures of the interior angles of a triangle is 180 degrees. It's given that the measure of $\angle SQX$ is 48° and the measure of $\angle SXQ$ is 86° . Since points S , Q , and X form a triangle, it follows from the triangle angle sum theorem that the measure, in degrees, of $\angle QSX$ is $180 - 48 - 86$, or 46. It's also given that the measure of $\angle SWU$ is 85° . Since $\angle SWU$ and $\angle SWR$ are supplementary angles, the sum of their measures is 180 degrees. It follows that the measure, in degrees, of $\angle SWR$ is $180 - 85$, or 95. Since points R , S , and W form a triangle, and $\angle RSW$ is the same angle as $\angle QSX$, it follows from the triangle angle sum theorem that the measure, in degrees, of $\angle WRS$ is $180 - 46 - 95$, or 39. It's given that the measure of $\angle VTU$ is 162° . Since $\angle VTU$ and $\angle STU$ are supplementary angles, the sum of their measures is 180 degrees. It follows that the measure, in degrees, of $\angle STU$ is $180 - 162$, or 18. Since points R , T , and U form a triangle, and $\angle URT$ is the same angle as $\angle WRS$, it follows from the triangle angle sum theorem that the measure, in degrees, of $\angle TUR$ is $180 - 39 - 18$, or 123.

Question a639d262

Geometry and Trigonometry - Lines, angles, and triangles

23/34

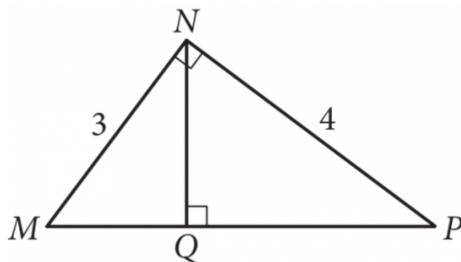
✂

🔖

📄

ABC

Show



In the figure above, what is the length of $\overline{NQ}$?

- ☐ 2.2
- ☐ 2.3
- ☒ 2.4
- ☐ 2.5

Choice C is correct. First, $\overline{MP}$ is the hypotenuse of right $\triangle MNP$, whose legs have lengths 3 and 4. Therefore, $(MP)^2 = 3^2 + 4^2$, so $(MP)^2 = 25$ and $MP = 5$. Second, because $\angle MNP$ corresponds to $\angle NQP$ and because $\angle MPN$ corresponds to $\angle NPQ$, $\triangle MNP$ is similar to $\triangle NQP$. The ratio of corresponding sides of similar triangles is constant, so $\frac{NQ}{MN} = \frac{NP}{MP}$. Since $MP = 5$ and it's given that $MN = 3$ and $NP = 4$, $\frac{NQ}{3} = \frac{4}{5}$. Solving for NQ results in $NQ = \frac{12}{5}$, or 2.4.

Question b8bbe611

Geometry and Trigonometry - Lines, angles, and triangles

26/34

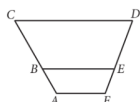
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ABC

Show



In the figure above, $\overline{AF}$, $\overline{BE}$, and $\overline{CD}$ are parallel. Points B and E lie on $\overline{AC}$ and $\overline{FD}$, respectively. If $AB = 9$, $BC = 18.5$, and $FE = 8.5$, what is the length of $\overline{ED}$, to the nearest tenth?

- ☐ 16.8
- ☐ 17.5
- ☐ 18.4
- ☐ 19.6

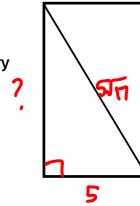
Choice B is correct. Since $\overline{AF}$, $\overline{BE}$, and $\overline{CD}$ are parallel, quadrilaterals $AFEB$ and $BEDC$ are similar. Let x represent the length of $\overline{ED}$. With similar figures, the ratios of the lengths of corresponding sides are equal. It follows that $\frac{9}{18.5} = \frac{8.5}{x}$. Multiplying both sides of this equation by 18.5 and by x yields $9x = (18.5)(8.5)$, or $9x = 157.25$. Dividing both sides of this equation by 9 yields $x = 17.47$, which to the nearest tenth is 17.5.

• Right triangles and trigonometry

Question 168ee60

Geometry and Trigonometry - Right triangles and trigonometry

1/23 Show



The length of a rectangle's diagonal is $5\sqrt{17}$, and the length of the rectangle's shorter side is 5. What is the length of the rectangle's longer side?

- ☐ $\sqrt{17}$
- ☐ 20
- ☐ $15\sqrt{2}$
- ☐ 400

Choice B is correct. A rectangle's diagonal divides a rectangle into two congruent right triangles, where the diagonal is the hypotenuse of both triangles. It's given that the length of the diagonal is $5\sqrt{17}$ and the length of the rectangle's shorter side is 5. Therefore, each of the two right triangles formed by the rectangle's diagonal has a hypotenuse with length $5\sqrt{17}$, and a shorter leg with length 5. To calculate the length of the longer leg of each right triangle, the Pythagorean theorem, $a^2 + b^2 = c^2$, can be used, where a and b are the lengths of the legs and c is the length of the hypotenuse of the triangle. Substituting 5 for a and $5\sqrt{17}$ for c in the equation $a^2 + b^2 = c^2$ yields $5^2 + b^2 = (5\sqrt{17})^2$, which is equivalent to $25 + b^2 = 25(17)$, or $25 + b^2 = 425$. Subtracting 25 from each side of this equation yields $b^2 = 400$. Taking the positive square root of each side of this equation yields $b = 20$. Therefore, the length of the longer leg of each right triangle formed by the diagonal of the rectangle is 20. It follows that the length of the rectangle's longer side is 20.

Choice A is incorrect and may result from dividing the length of the rectangle's diagonal by the length of the rectangle's shorter side, rather than substituting these values into the Pythagorean theorem.

Choice C is incorrect and may result from using the length of the rectangle's diagonal as the length of a leg of the right triangle, rather than the length of the hypotenuse.

Choice D is incorrect. This is the square of the length of the rectangle's longer side.

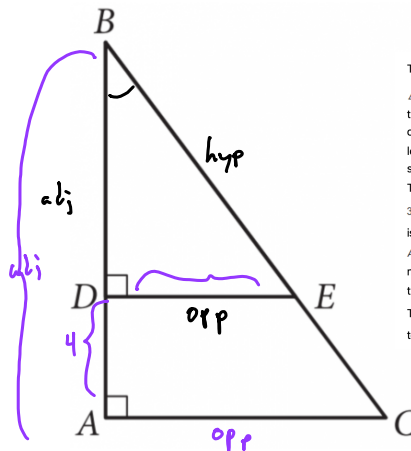
sol cal for right Δs

$$\tan E = \frac{\text{opp}}{\text{adj}}$$

Question c2d89f4

Geometry and Trigonometry - Right triangles and trigonometry

2/23 Show



The correct answer is 6. Since $\tan B = \frac{3}{4}$, $\triangle ABC$ and $\triangle DBE$ are both similar to 3-4-5 triangles. This means that they are both similar to the right triangle with sides of lengths 3, 4, and 5. Since $BC = 15$, which is 3 times as long as the hypotenuse of the 3-4-5 triangle, the similarity ratio of $\triangle ABC$ to the 3-4-5 triangle is 3:1. Therefore, the length of $\overline{AC}$ (the side opposite to $\angle B$) is $3 \times 3 = 9$, and the length of $\overline{AB}$ (the side adjacent to $\angle B$) is $4 \times 3 = 12$. It is also given that $DA = 4$. Since $AB = DA + DB$ and $AB = 12$, it follows that $DB = 8$, which means that the similarity ratio of $\triangle DBE$ to the 3-4-5 triangle is 2:1 ($\overline{DB}$ is the side adjacent to $\angle B$). Therefore, the length of $\overline{DE}$, which is the side opposite to $\angle B$, is $3 \times 2 = 6$.

$$\frac{DE}{BD} = \frac{3}{4}$$

$$\frac{AC}{BA} = \frac{3}{4}$$

In the figure above, $\tan B = \frac{3}{4}$. If $BC = 15$ and $DA = 4$, what is the length of $\overline{DE}$?

Answer



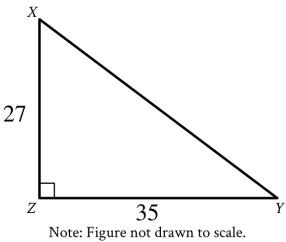
Question 120f3d69

Geometry and Trigonometry - Right triangles and trigonometry

3/23

ABC

Show



Triangle XYZ shown is a right triangle. Which of the following has the same value as $\sin X$?

- ☐ $\tan X$

×
- ☐ $\tan Y$

×
- ☐ $\cos X$
- ☐ $\cos Y$

Choice D is correct. The sine of an angle is equal to the cosine of its complementary angle. In the triangle shown, angle Z is a right angle; thus, angles X and Y are complementary angles. Therefore, $\cos Y$ has the same value as $\sin X$.

Question e362d5ab

Geometry and Trigonometry - Right triangles and trigonometry

21/23

ABC

Show

For two acute angles, $\angle Q$ and $\angle R$, $\cos(Q) = \sin(R)$. The measures, in degrees, of $\angle Q$ and $\angle R$ are $x + 61$ and $4x + 4$, respectively. What is the value of x ?

- ☐ 5
- ☐ 19
- ☐ 23
- ☐ 29

Choice A is correct. It's given that for two acute angles, $\angle Q$ and $\angle R$, $\cos(Q) = \sin(R)$. For two acute angles, if the sine of one angle is equal to the cosine of the other angle, the angles are complementary. It follows that $\angle Q$ and $\angle R$ are complementary. That is, the sum of the measures of the angles is 90 degrees. It's given that the measure of $\angle Q$ is $x + 61$ degrees and the measure of $\angle R$ is $4x + 4$ degrees. It follows that $(x + 61) + (4x + 4) = 90$. By combining like terms, this equation can be rewritten as $5x + 65 = 90$. Subtracting 65 from each side of this equation yields $5x = 25$. Dividing each side of this equation by 5 yields $x = 5$.

Choice B is incorrect. This would be the value of x if $\cos(Q) = \cos(R)$ rather than $\cos(Q) = \sin(R)$.

Choice C is incorrect. This would be the value of x if $\cos(Q) = -\cos(R)$ rather than $\cos(Q) = \sin(R)$ and if $\angle R$ were obtuse rather than acute.

Choice D is incorrect and may result from conceptual or calculation errors.

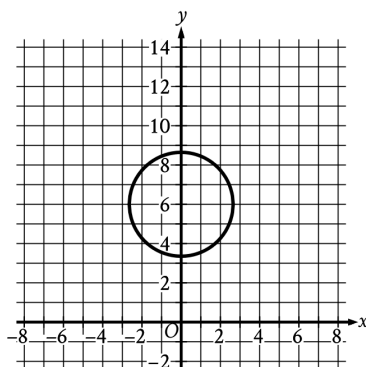
• Circles

Question 1b2b20b9
Geometry and Trigonometry - Circles

1/27



Show



The correct answer is 28. The equation of a circle in the xy -plane can be written as $(x - t)^2 + (y - s)^2 = r^2$, where the center of the circle is (t, s) and the radius of the circle is r . It's given that circle A is defined by the equation $x^2 + (y - 6)^2 = 7$, which can be written as $(x - 0)^2 + (y - 6)^2 = (\sqrt{7})^2$. It follows that $r = \sqrt{7}$ and the radius of circle A is $\sqrt{7}$. It's also given that circle B has the same radius as circle A. If the equation of circle B is $(x - h)^2 + (y - k)^2 = a$, then $a = r^2$. Substituting $\sqrt{7}$ for r in this equation yields $a = (\sqrt{7})^2$, or $a = 7$. It follows that the value of $4a$ is $4(7)$, or 28.

Circle A shown is defined by the equation $x^2 + (y - 6)^2 = 7$. Circle B (not shown) has the same radius but is translated 96 units to the right. If the equation of circle B is $(x - h)^2 + (y - k)^2 = a$, where h , k , and a are constants, what is the value of $4a$?

Answer



Question 2266984b
Geometry and Trigonometry - Circles

2/27



Show

$$x^2 + 20x + y^2 + 16y = -20$$

The equation above defines a circle in the xy -plane. What are the coordinates of the center of the circle?

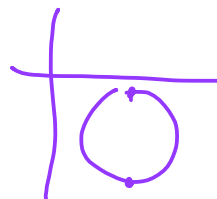
☐ $(-20, -16)$



☐ $(-10, -8)$

☐ $(10, 8)$

☐ $(20, 16)$



Choice B is correct. The standard equation of a circle in the xy -plane is of the form $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) are the coordinates of the center of the circle and r is the radius. The given equation can be rewritten in standard form by completing the squares. So the sum of the first two terms, $x^2 + 20x$, needs a 100 to complete the square, and the sum of the second two terms, $y^2 + 16y$, needs a 64 to complete the square. Adding 100 and 64 to both sides of the given equation yields $(x^2 + 20x + 100) + (y^2 + 16y + 64) = -20 + 100 + 64$, which is equivalent to $(x + 10)^2 + (y + 8)^2 = 144$. Therefore, the coordinates of the center of the circle are $(-10, -8)$.

Question 2855cb58

Geometry and Trigonometry - Circles

3/27



Show

A circle in the xy -plane has its center at $(16, 17)$ and has a radius of $7k$. Which equation represents this circle?

☐ $(x - 16)^2 + (y - 17)^2 = 49k$

✕

☐ $(x - 16)^2 + (y - 17)^2 = 49k^2$

☐ $(x - 16)^2 + (y - 17)^2 = 7k$

☐ $(x - 16)^2 + (y - 17)^2 = 7k^2$

Choice B is correct. The equation of a circle in the xy -plane can be written as $(x - h)^2 + (y - k)^2 = r^2$, where the center of the circle is (h, k) and the radius of the circle is r . It's given that this circle has a center at $(16, 17)$ and a radius of $7k$. Substituting 16 for h , 17 for k , and $7k$ for r in $(x - h)^2 + (y - k)^2 = r^2$ yields $(x - 16)^2 + (y - 17)^2 = (7k)^2$, or $(x - 16)^2 + (y - 17)^2 = 49k^2$. Therefore, the equation that represents this circle is $(x - 16)^2 + (y - 17)^2 = 49k^2$.

Choice A is incorrect. This equation represents a circle with radius $7\sqrt{k}$, not $7k$.

Choice C is incorrect. This equation represents a circle with radius $\sqrt{7k}$, not $7k$.

Choice D is incorrect. This equation represents a circle with radius $\sqrt{7k}$, not $7k$.

Question 2d521ca9

Geometry and Trigonometry - Circles

4/27



Show

The measure of angle Z is 60° . What is the measure, in radians, of angle Z ?

☐ $\frac{1}{6}\pi$

☐ $\frac{1}{3}\pi$

☐ $\frac{2}{3}\pi$

☐ 1π

Choice B is correct. The measure of an angle, in radians, can be found by multiplying its measure, in degrees, by $\frac{\pi}{180}$. It's given that the measure of angle Z is 60° . It follows that the measure, in radians, of angle Z is $60\left(\frac{\pi}{180}\right)$, or $\frac{1}{3}\pi$.

Choice A is incorrect. This is the measure, in radians, of an angle whose measure is 30° , not 60° .

Choice C is incorrect. This is the measure, in radians, of an angle whose measure is 120° , not 60° .

Choice D is incorrect. This is the measure, in radians, of an angle whose measure is 180° , not 60° .

$$\frac{2\pi \text{ radians} = 360^\circ}{? \quad 60^\circ}$$

Question 74d8b897

Geometry and Trigonometry - Circles

8/27



Show

An angle has a measure of $\frac{9\pi}{20}$ radians. What is the measure of the angle in degrees?

Answer

The correct answer is 81. The measure of an angle, in degrees, can be found by multiplying its measure, in radians, by $\frac{180 \text{ degrees}}{\pi \text{ radians}}$. Multiplying the given angle measure, $\frac{9\pi}{20}$ radians, by $\frac{180 \text{ degrees}}{\pi \text{ radians}}$ yields $(\frac{9\pi}{20} \text{ radians}) (\frac{180 \text{ degrees}}{\pi \text{ radians}})$, which is equivalent to 81 degrees.

Question 35d37640

Geometry and Trigonometry - Circles

5/27



Show

Point F lies on a unit circle in the xy -plane and has coordinates $(1, 0)$. Point G is the center of the circle and has coordinates $(0, 0)$. Point H also lies on the circle and has coordinates $(-1, y)$, where y is a constant. Which of the following could be the positive measure of angle FGH , in radians?

☐ $\frac{27\pi}{2}$

☐ $\frac{29\pi}{2}$

☐ 24π

☐ 25π

Choice D is correct. It's given that the circle is a unit circle, which means the circle has a radius of 1. It's also given that point G is the center of the circle and has coordinates $(0, 0)$ and that point H lies on the circle and has coordinates $(-1, y)$. Since the radius of the circle is 1, the value of y must be 0, as all other points with an x -coordinate of -1 are a distance greater than 1 away from point G . Since F and H are points on the unit circle centered at G , let side FG be the starting side of the angle and side GH be the terminal side of the angle. Since side FG is on the positive x -axis and side GH is on the negative x -axis, side FG is half of a rotation around the unit circle, or π radians, away from side GH . Therefore, the positive measure of angle FGH , in radians, is equal to π plus an integer multiple of 2π . In other words, the positive measure of angle FGH , in radians, is an odd integer multiple of π . Of the given choices, only 25π is an odd integer multiple of π .

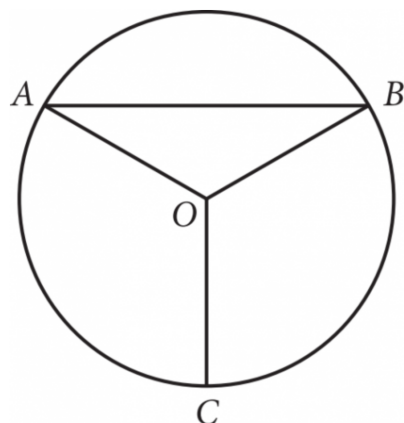
Choice A is incorrect. This could be the positive measure of an angle where the starting side is FG and the terminal side contains the point $(0, -1)$, not $(-1, 0)$.

Choice B is incorrect. This could be the positive measure of an angle where the starting side is FG and the terminal side contains the point $(0, 1)$, not $(-1, 0)$.

Choice C is incorrect. This could be the positive measure of an angle where the starting side is FG and the terminal side contains the point $(1, 0)$, not $(-1, 0)$.

Question 69b0d79d ■ ■ ■
Geometry and Trigonometry - Circles

7/27 Show



Point O is the center of the circle above, and the measure of $\angle OAB$ is 30° . If the length of $\overline{OC}$ is 18, what is the length of arc $\widehat{AB}$?

- ☐ 9π ×
- ☐ 12π ×
- ☐ 15π ×
- ☐ 18π ×

Choice B is correct. Because segments OA and OB are radii of the circle centered at point O, these segments have equal lengths. Therefore, triangle AOB is an isosceles triangle, where angles OAB and OBA are congruent base angles of the triangle. It's given that angle OAB measures 30° . Therefore, angle OBA also measures 30° . Let x° represent the measure of angle AOB. Since the sum of the measures of the three angles of any triangle is 180° , it follows that $30^\circ + 30^\circ + x^\circ = 180^\circ$, or $60^\circ + x^\circ = 180^\circ$. Subtracting 60° from both sides of this equation yields $x^\circ = 120^\circ$, or $\frac{2\pi}{3}$ radians. Therefore, the measure of angle AOB, and thus the measure of arc $\widehat{AB}$, is $\frac{2\pi}{3}$ radians. Since $\overline{OC}$ is a radius of the given circle and its length is 18, the length of the radius of the circle is 18. Therefore, the length of arc $\widehat{AB}$ can be calculated as $\left(\frac{2\pi}{3}\right)(18)$, or 12π .

Question 3e577e4a ■ ■ ■
Geometry and Trigonometry - Circles

6/27 Show

A circle in the xy -plane has its center at $(-4, -6)$. Line k is tangent to this circle at the point $(-7, -7)$. What is the slope of line k ?

- ☐ -3
- ☐ $-\frac{1}{3}$
- ☐ $\frac{1}{3}$
- ☐ 3

Choice A is correct. A line that's tangent to a circle is perpendicular to the radius of the circle at the point of tangency. It's given that the circle has its center at $(-4, -6)$ and line k is tangent to the circle at the point $(-7, -7)$. The slope of a radius defined by the points (q, r) and (s, t) can be calculated as $\frac{t-r}{s-q}$. The points $(-7, -7)$ and $(-4, -6)$ define the radius of the circle at the point of tangency. Therefore, the slope of this radius can be calculated as $\frac{(-6)-(-7)}{(-4)-(-7)}$, or $\frac{1}{3}$. If a line and a radius are perpendicular, the slope of the line must be the negative reciprocal of the slope of the radius. The negative reciprocal of $\frac{1}{3}$ is -3 . Thus, the slope of line k is -3 .

Question 82c8325f

Geometry and Trigonometry - Circles

10/27



Show

A circle in the xy -plane has its center at $(-4, 5)$ and the point $(-8, 8)$ lies on the circle. Which equation represents this circle?

☐ $(x - 4)^2 + (y + 5)^2 = 5$

☐ $(x + 4)^2 + (y - 5)^2 = 5$

☐ $(x - 4)^2 + (y + 5)^2 = 25$

☐ $(x + 4)^2 + (y - 5)^2 = 25$

Choice D is correct. A circle in the xy -plane can be represented by an equation of the form $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center of the circle and r is the length of a radius of the circle. It's given that the circle has its center at $(-4, 5)$. Therefore, $h = -4$ and $k = 5$. Substituting -4 for h and 5 for k in the equation $(x - h)^2 + (y - k)^2 = r^2$ yields $(x - (-4))^2 + (y - 5)^2 = r^2$, or $(x + 4)^2 + (y - 5)^2 = r^2$. It's also given that the point $(-8, 8)$ lies on the circle. Substituting -8 for x and 8 for y in the equation $(x + 4)^2 + (y - 5)^2 = r^2$ yields $(-8 + 4)^2 + (8 - 5)^2 = r^2$, or $(-4)^2 + (3)^2 = r^2$, which is equivalent to $16 + 9 = r^2$, or $25 = r^2$. Substituting 25 for r^2 in the equation $(x + 4)^2 + (y - 5)^2 = r^2$ yields $(x + 4)^2 + (y - 5)^2 = 25$. Thus, the equation $(x + 4)^2 + (y - 5)^2 = 25$ represents the circle.

Question 89661424

Geometry and Trigonometry - Circles

12/27

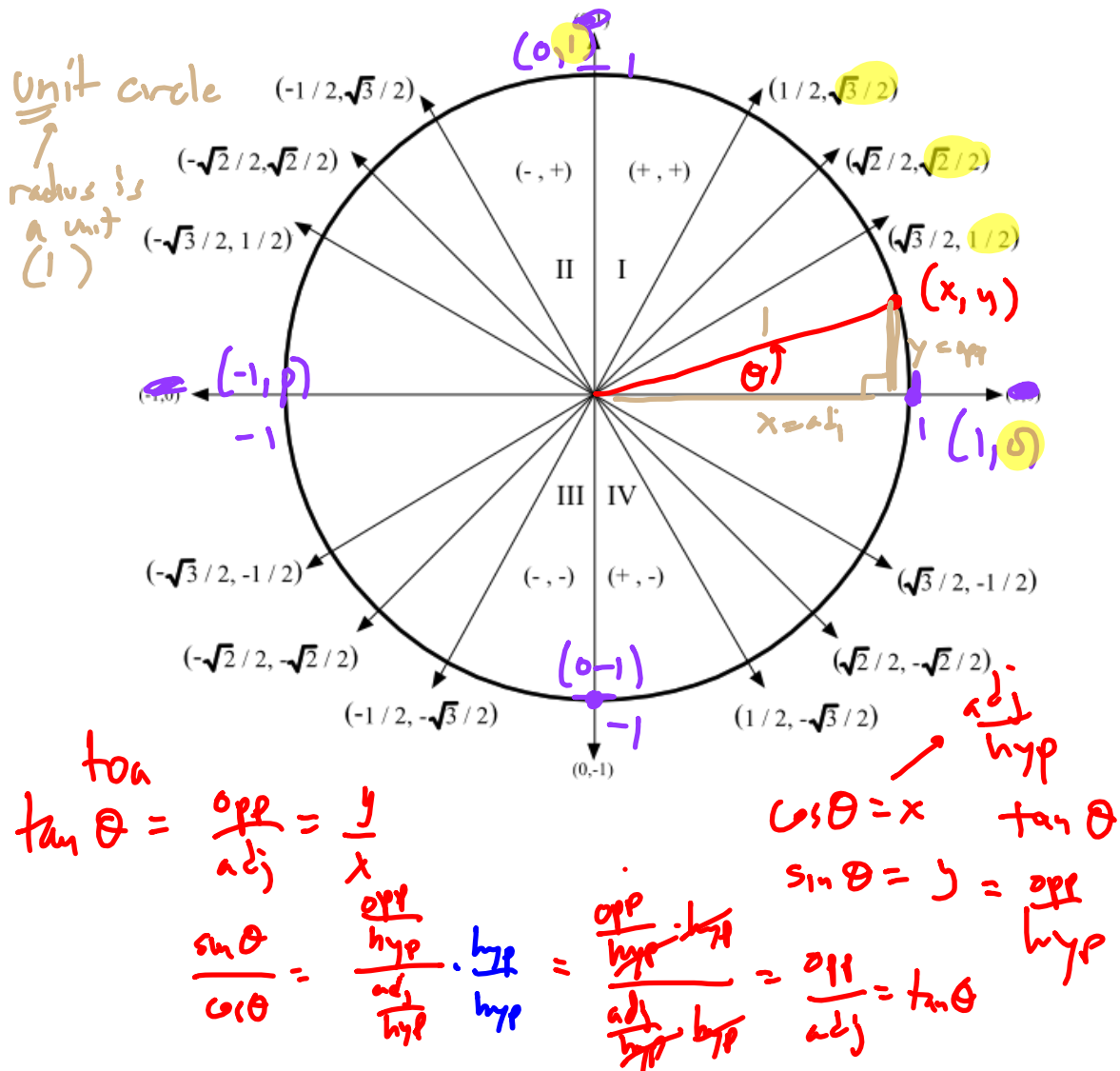


Show

A circle in the xy -plane has its center at $(-5, 2)$ and has a radius of 9. An equation of this circle is $x^2 + y^2 + ax + by + c = 0$, where a , b , and c are constants. What is the value of c ?

Answer

The correct answer is -52 . The equation of a circle in the xy -plane with its center at (h, k) and a radius of r can be written in the form $(x - h)^2 + (y - k)^2 = r^2$. It's given that a circle in the xy -plane has its center at $(-5, 2)$ and has a radius of 9. Substituting -5 for h , 2 for k , and 9 for r in the equation $(x - h)^2 + (y - k)^2 = r^2$ yields $(x - (-5))^2 + (y - 2)^2 = 9^2$, or $(x + 5)^2 + (y - 2)^2 = 81$. It's also given that an equation of this circle is $x^2 + y^2 + ax + by + c = 0$, where a , b , and c are constants. Therefore, $(x + 5)^2 + (y - 2)^2 = 81$ can be rewritten in the form $x^2 + y^2 + ax + by + c = 0$. The equation $(x + 5)^2 + (y - 2)^2 = 81$, or $(x + 5)(x + 5) + (y - 2)(y - 2) = 81$, can be rewritten as $x^2 + 5x + 5x + 25 + y^2 - 2y - 2y + 4 = 81$. Combining like terms on the left-hand side of this equation yields $x^2 + y^2 + 10x - 4y + 29 = 81$. Subtracting 81 from both sides of this equation yields $x^2 + y^2 + 10x - 4y - 52 = 0$, which is equivalent to $x^2 + y^2 + 10x + (-4)y + (-52) = 0$. This equation is in the form $x^2 + y^2 + ax + by + c = 0$. Therefore, the value of c is -52 .



θ	$\sin \theta$	$\cos \theta$
0	$\frac{\sqrt{0}}{2} = \frac{0}{2} = 0$	$\frac{\sqrt{4}}{2} = 1$
$\frac{\pi}{6} = 30^\circ$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4} = 45^\circ$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3} = 60^\circ$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{2} = 90^\circ$	$\frac{\sqrt{4}}{2} = \frac{2}{2} = 1$	$\frac{\sqrt{0}}{2} = 0$

$$n \cdot n = 0$$

$$\sqrt{0} = 0$$

$$\int_0^{\frac{\pi}{6}} \sin x \, dx = \left[-\cos x \right]_0^{\frac{\pi}{6}}$$

$$-\cos \frac{\pi}{6} - (-\cos 0)$$

$$-\cos \frac{\pi}{6} + \cos 0$$

$$-\frac{\sqrt{3}}{2} + 1$$

