

Outline of IB Mathematics SL syllabus, first exams 2014

1 Algebra

- 1.1 Arithmetic sequences and series; sum of finite arithmetic series; geometric sequences and series; sum of finite and infinite geometric series. Sigma notation. Applications. Examples include compound interest and population growth.
- 1.2 Elementary treatment of exponents and logarithms. Laws of exponents; laws of logarithms. Change of base.
- 1.3 The binomial theorem: expansion of $(a + b)^n$, $n \in \mathbb{N}$. Calculation of binomial coefficients using Pascal's triangle and $\binom{n}{r} \cdot \binom{n}{r}$ should be found using both the formula and technology.

2 Functions and equations

- 2.1 Concept of function $f: x \mapsto f(x)$. Domain, range; image (value). Composite functions. Identity function. Inverse function f^{-1} .
- 2.2 The graph of a function; its equation $y = f(x)$. Function graphing skills. Investigation of key features of graphs, such as maximum and minimum values, intercepts, horizontal and vertical asymptotes, symmetry, and consideration of domain and range. Use of technology to graph a variety of functions, including ones not specifically mentioned. The graph of $y = f^{-1}(x)$ as the reflection in the line $y = x$ of the graph of $y = f(x)$. Note the difference in the command terms “draw” and “sketch”.
- 2.3 Transformations of graphs. Translations: $y = f(x) + b$; $y = f(x - a)$. Reflections (in both axes): $y = -f(x)$; $y = f(-x)$. Vertical stretch with scale factor p : $y = pf(x)$. Stretch in the x -direction with scale factor $\frac{1}{q}$: $y = f(qx)$. Composite transformations. Note: translation by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ denotes horizontal shift of 3 units to the right and vertical shift of 2 down.
- 2.4 The quadratic function $x \mapsto ax^2 + bx + c$: its graph, y -intercept $(0, c)$. Axis of symmetry. The form $x \mapsto a(x - p)(x - q)$, x -intercepts $(p, 0)$ and $(q, 0)$. The form $x \mapsto a(x - h)^2 + k$, vertex (h, k) . Candidates are expected to be able to change from one form to another.
- 2.5 The reciprocal function $x \mapsto \frac{1}{x}$, $x \neq 0$: its graph and self-inverse nature. The rational function $x \mapsto \frac{ax + b}{cx + d}$ and its graph. Vertical and horizontal asymptotes. Diagrams should include all asymptotes and intercepts.
- 2.6 Exponential functions and their graphs: $x \mapsto a^x$, $a > 0$, $x \mapsto e^x$. Logarithmic functions and their graphs: $x \mapsto \log_a x$, $x > 0$, $x \mapsto \ln x$, $x > 0$. Relationships between these functions: $a^x = e^{x \ln a}$; $\log_a a^x = x$; $a^{\log_a x} = x$, $x > 0$.
- 2.7 Solving equations, both graphically and analytically. Solutions may be referred to as roots of equations or zeros of functions. Use of technology to solve a variety of equations, including those where there is no appropriate analytic approach. Solving $ax^2 + bx + c = 0$, $a \neq 0$. The quadratic formula. The discriminant $\Delta = b^2 - 4ac$ and the nature of the roots, that is, two distinct real roots, two equal real roots, no real roots. Solving exponential equations.
- 2.8 Applications of graphing skills and solving equations that relate to real-life situations.

3 Circular functions and trigonometry

- 3.1 The circle: radian measure of angles; length of an arc; area of a sector. Radian measure may be expressed as exact multiples of π or as decimals.

- 3.2 Definition of $\cos \theta$ and $\sin \theta$ in terms of the unit circle. Definition of $\tan \theta$ as $\frac{\sin \theta}{\cos \theta}$. Exact values of trigonometric ratios of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ and their multiples. The equation of a straight line through the origin is $y = x \tan \theta$.
- 3.3 The Pythagorean identity $\cos^2 \theta + \sin^2 \theta = 1$. Double angle identities for sine and cosine. Relationship between trigonometric ratios.
- 3.4 The circular functions $\sin x, \cos x$, and $\tan x$: their domains and ranges; amplitude, their periodic nature; and their graphs. Composite functions of the form $f(x) = a \sin(b(x + c)) + d$. Transformations. Applications.
- 3.5 Solving trigonometric equations in a finite interval, both graphically and analytically. Equations leading to quadratic equations in $\sin x, \cos x$, or $\tan x$.
- 3.6 Solution of triangles. The cosine rule. The sine rule, including the ambiguous case. Area of a triangle, $\frac{1}{2} ab \sin C$. Applications. Examples include navigation, problems in two and three dimensions, including angles of elevation and depression.

4 Vectors

- 4.1 Vectors as displacements in the plane and in three dimensions. Components of a vector; column representation; $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1 \mathbf{i} + v_2 \mathbf{j} + v_3 \mathbf{k}$. Algebraic and geometric approaches to the following: the sum and difference of two vectors; the zero vector, the vector $-\mathbf{v}$; multiplication by a scalar, $k\mathbf{v}$; parallel vectors; magnitude of a vector, $|\mathbf{v}|$; unit vectors; base vectors \mathbf{i}, \mathbf{j} , and \mathbf{k} ; position vectors $\overrightarrow{OA} = \mathbf{a}$; $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \mathbf{b} - \mathbf{a}$.
- 4.2 The scalar product (dot product) of two vectors. Perpendicular vectors; parallel vectors. The angle between two vectors. For non-zero vectors, $\mathbf{v} \cdot \mathbf{w} = 0$ is equivalent to the vectors being perpendicular. For parallel vectors, $\mathbf{w} = k\mathbf{v}$, $|\mathbf{v} \cdot \mathbf{w}| = |\mathbf{v}||\mathbf{w}|$.
- 4.3 Vector equation of a line in two and three dimensions: $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. The angle between two lines. Relevance of \mathbf{a} (position) and \mathbf{b} (direction). Interpretation of t as time and \mathbf{b} as velocity, with $|\mathbf{b}|$ representing speed.
- 4.4 Distinguishing between coincident and parallel lines. Finding the point of intersection of two lines. Determining whether two lines intersect.

5 Statistics and probability

- 5.1 Concepts of population, sample, random sample, discrete and continuous data. Presentation of data: frequency distributions (tables); frequency histograms with equal class intervals; box-and-whisker plots; outliers. Grouped data: use of mid-interval values for calculations; interval width; upper and lower interval boundaries; modal class. Outliers are defined as more than $1.5 \times \text{IQR}$ from the nearest quartile.
- 5.2 Statistical measures and their interpretations. Central tendency: mean, median, mode. Quartiles, percentiles. Dispersion: range, interquartile range, variance, standard deviation. Effect of constant changes to the original data. Applications.
- 5.3 Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles.
- 5.4 Linear correlation of bivariate data. Pearson's product-moment correlation coefficient r . Scatter diagrams; lines of best fit. Equation of the regression line of y on x . Use of the equation for prediction purposes. Mathematical and contextual interpretation. Validity of interpolation versus extrapolation.

- 5.5 Concepts of trial, outcome, equally likely outcomes, sample space (U) and event. The probability of an event A is $P(A) = \frac{n(A)}{n(U)}$. The complementary events A and A' (not A). Use of Venn diagrams, tree diagrams and tables of outcomes.
- 5.6 Combined events, $P(A \cup B)$. The non-exclusivity of “or.” Mutually exclusive events: $P(A \cap B) = 0$. Conditional probability; the definition $P(A|B) = \frac{P(A \cap B)}{P(B)}$. Independent events; the definition $P(A|B) = P(A) = P(A|B')$. Probabilities with and without replacement.
- 5.7 Concept of discrete random variables and their probability distributions. Expected values (mean), $E(X)$ for discrete data. Applications, including games of chance.
- 5.8 Binomial distribution. Mean and variance of the binomial distribution. Conditions under which random variables have this distribution.
- 5.9 Normal distributions and curves. Standardization of normal variables (z -values, z -scores). Properties of the normal distribution.

6 Calculus

- 6.1 Informal ideas of limit and convergence. Limit notation. Definition of derivative from first principles as $f'(x) = \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right)$. Derivative interpreted as gradient (slope) function and as rate of change. Tangents and normals and their equations. Use of both forms of notation, $\frac{dy}{dx}$ and $f'(x)$, for the first derivative. Identify intervals on which functions are increasing or decreasing.
- 6.2 Derivative of x^n ($n \in \mathbb{Q}$), $\sin x$, $\cos x$, $\tan x$, e^x and $\ln x$. Differentiation of a sum and a real multiple of these functions. The chain rule for composite functions. The product and quotient rules. The second derivative. Extension to higher derivatives.
- 6.3 Local maximum and minimum points. Testing for maximum or minimum using change of sign of first derivative and sign of second derivative. Use of the terms “concave up” and “concave down.” Points of inflexion with zero and non-zero gradients. At a point of inflexion, $f''(x) = 0$ and changes sign. $f''(x) = 0$ is not a sufficient condition for a point of inflexion. Graphical behaviour of functions, including the relationship between the graphs of f , f' , and f'' . Optimization. Applications.
- 6.4 Indefinite integration as anti-differentiation. Indefinite integral of x^n ($n \in \mathbb{Q}$), $\sin x$, $\cos x$, $\frac{1}{x}$ and e^x . The composites of any of these with the linear function $ax + b$. Integration by inspection, or substitution of the form $\int f(g(x))g'(x)dx$.
- 6.5 Anti-differentiation with a boundary condition to determine the constant term. Definite integrals, both analytically and using technology. Areas under curves (between the curve and the x -axis). Areas between curves. Volumes of revolution about the x -axis. The values of some definite integrals can only be found using technology. Students are expected to first write a correct expression before calculating the area or volume.
- 6.6 Kinematic problems involving displacement s , velocity v , and acceleration a . Total distance travelled, $\int_{t_1}^{t_2} |v| dt$.