Practice Exam AB-1

E 1. $f(x) = e^{2x}\tan^{-1}(x)$. Find $f'(1)$

$$f'(x) = e^{2x}\cdot\frac{1}{1+x^2} + \tan^{-1}(x)\cdot 2e^{2x}$$

$$f'(1) = \frac{e^2}{1+1} + \tan^{-1}(1)\cdot 2e^2 = \frac{1}{2}e^2 + \frac{\pi}{4}\cdot 2e^2$$

E 2. $$\int_1^8 x^{-2/3}\,dx = 3x^{1/3}\Big]_1^8 = 3(8^{1/3} - 1^{1/3}) = 3(2-1) = 3$$

E 3. $f(x) = e^{-x} + \sin x - \cos x$. Find $f''(0)$.

$$f'(x) = -e^{-x} + \cos x + \sin x$$

$$f''(x) = e^{-x} - \sin x + \cos x$$

$$f''(0) = 1 - 0 + 1 = 2$$

D 4. $3x^2 - 2xy + y^2 = 11$ Find slope at $(1,-2)$.

$$6x - 2x\frac{dy}{dx} + y\cdot -2 + 2y\cdot\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-6x+2y}{-2x+2y}$$ when ~~x~~ at $(1,-2)$, gives

$$\frac{dy}{dx} = \frac{-6+-4}{-2-4} = \frac{-10}{-6} = \frac{5}{3}$$

B 5. $$\frac{d}{dx}\left(\ln(\sec x)\right) = \frac{1}{\sec x}\cdot \sec x\tan x = \tan x$$

B 6. $\int_{\pi/6}^{\pi/4} 2\sin(2x)\cos(2x)\,dx$

$u = \sin 2x$, so $du = 2\cos 2x$

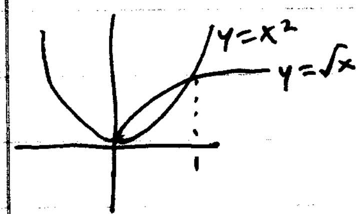
$x = \frac{\pi}{6} \Rightarrow u = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{4} \rightarrow u = \sin\frac{\pi}{2} = 1$

$= \int_{\sqrt{3}/2}^{1} u\,du$

$= \frac{1}{2}u^2\Big]_{\frac{\sqrt{3}}{2}}^{1} = \frac{1}{2}\left(1^2 - \left(\frac{\sqrt{3}}{2}\right)^2\right) = \frac{1}{2}\left(1 - \frac{3}{4}\right) = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8}$

B 7.



$\int_0^1 (\sqrt{x} - x^2)\,dx =$

$\left[\frac{2}{3}x^{3/2} - \frac{1}{3}x^3\right]_0^1$

$= \left(\frac{2}{3} - \frac{1}{3}\right) - (0 - 0) = \frac{1}{3}$

B 8. $f(x) = 2x \quad g(x) = x^2 \quad h(x) = 2^x$

2^x grows fastest, so $h(x)$ in denominator

A 9. $\lim_{x\to 5}\left(\frac{2x^2 - 50}{x^2 - 15x + 50}\right) \qquad \frac{50 - 50}{25 - 75 + 50} = \frac{0}{0}$

L'HÔ: $\lim_{x\to 5} \frac{4x}{2x - 15} = \frac{20}{10 - 15} = \frac{20}{-5} = -4$

A 10. $f''(0) < 0$ cc down $\qquad f'(0) = 0$, h. tangent $\qquad f(0) > 0$ positive y-int.

E 11. $\int_0^3 e^{\sin x}\,dx = k$ Find $\int_1^2 x\, e^{\sin(4-x^2)}\,dx$

Let $u = 4-x^2$, so $du = -2x\,dx$

$-\frac{1}{2}\int_1^2 -2x\, e^{\sin(4-x^2)}\,dx$

$x=1 \Rightarrow u = 4-1=3$

$x=2 \Rightarrow u = 4-4=0$

$= -\frac{1}{2}\int_3^0 e^{\sin u}\,du = \frac{1}{2}\int_0^3 e^{\sin u}\,du = \frac{1}{2}k$

B 12. $a(t) = t^2$ ft/s^2 $v(0) = -72$ ft/s When does it change direction?

Need to know when $v(t)=0$.

$v(t) = \int t^2\,dt = \frac{1}{3}t^3 + C$. Since $v(0) = -72$, $C = -72$.

$v(t) = \frac{1}{3}t^3 + -72 = 0$

$t^3 = 72 \cdot 3 = 216$

$t = 6$, And since v is cubic, it does change signs.

C 13. $y = x^3 + 3x^2 - 45x + 81$

$y' = 3x^2 + 6x - 45$

$y'' = 6x + 6$

p.o.i. when y'' changes signs:

$6x + 6 = 0$

$x = -1$

B 14. ~~0 = f''~~ $0 < f''(x) < f'(x) < f(x)$ for all x

Not A, b/c $f'(x) = -e^{-x} < 0$.

Not E, b/c $f'(x) = 2xe^{x^2}$, which is negative for $x < 0$.

Not C, b/c $f(x) = f'(x) = f''(x) = e^x$.

Try B: $f(x) = e^{x/2}$, $f'(x) = \frac{1}{2}e^{x/2}$, $f''(x) = \frac{1}{4}e^{x/2}$

Yep, it's B.

A 15. $f(-3) = 4$, $f'(-3) = 2$. Tangent line approx. of $f(-3.1)$

$$y - 4 = 2(x - (-3))$$

$$y = 2x + 6 + 4 = 2x + 10$$

$$y(-3.1) = 2(-3.1) + 10 = -6.2 + 10 = 3.8 \approx f(-3.1)$$

C 16. $f(x) = \sqrt{x-1}$ Avg. value on $[1, 5]$

$$\frac{1}{5-1}\int_1^5 \sqrt{x-1}\,dx = \frac{1}{4}\cdot\frac{2}{3}(x-1)^{3/2}\Big]_1^5$$

$$= \frac{1}{6}\left(4^{3/2} - 0^{3/2}\right) = \frac{1}{6}\cdot 8 = \frac{4}{3}$$

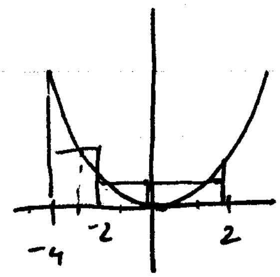
B 17. At $x=1$, $f' < 0$, and f' changes from decreasing to increasing $\Rightarrow$ point of inflection

A 18. f has maximum when f' changes $+$ to $-$, only once

C 19. $f(-2) = f(2) + \int_2^{-2} f'(x)\,dx = 1 - \int_{-2}^{2} f'(x)\,dx = 1 - (1 + \frac{1}{2} + -1) = \frac{1}{2}$

C 20. $\int_{-4}^{2} x^2\,dx$

mid point x-values are $-3, -1, 1$.



$\approx 2\cdot(-3)^2 + 2(-1)^2 + 2(1^2) = 2\cdot 9 + 2\cdot 1 + 2\cdot 1 = 22$

D 21. Slope $= 1$ when ~~y=0~~ $y=0 \Rightarrow \frac{dy}{dx} = 1+y^2$

Just in case: $\int \frac{dy}{1+y^2} = \int dx$

$\arctan y = x + c$

$y = \tan(x+c)$ yep.

D 22. $v(t) = 4 - t^2$ ft/s

$v(t) = 0$ at $t = \pm 2$

Distance on $t\in[0,2]$ is $\int_0^2 (4-t^2)\,dt = \left[4t - \frac{1}{3}t^3\right]_0^2$

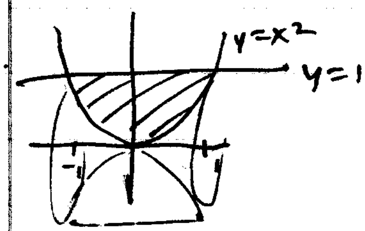
$= \left(8 - \frac{8}{3}\right) - 0 = \frac{16}{3}$

Distance on $t\in[2,3]$ is $\left|\int_2^3 (4-t^2)\,dt\right| = \left[4t - \frac{1}{3}t^3\right]_2^3$

$= \left|(12-9) - \left(8-\frac{8}{3}\right)\right| = \left|3 - \frac{16}{3}\right| = \frac{7}{3}$

Total distance $= \frac{16}{3} + \frac{7}{3} = \frac{23}{3}$

E 23.



$$\pi \int_{-1}^{1} (1^2-(x^2)^2)\, dx$$

$$= \pi \int_{-1}^{1} (1-x^4)dx = \pi \left(x - \tfrac{1}{5}x^5\right)\Big]_{-1}^{1}$$

$$= \pi\left(\left(1-\tfrac{1}{5}\right)-\left(-1+\tfrac{1}{5}\right)\right) = \tfrac{8}{5}\pi$$

C 24. $\frac{dy}{dx} = -4y$, $f(0)=6$

$$\int \frac{dy}{y} = \int -4\,dx$$

$$\ln|y| = -4x + C$$

$y = Ce^{-4x}$ and $C=6$

D 25. $F(x) = \int_0^{x^2} \cos(t^3)\,dt$

$$F'(x) = \cos(x^6)\cdot 2x$$

C 26. $\int_1^e \frac{\ln x}{x}\,dx$ $\qquad u = \ln x,\ du = \frac{1}{x}dx$

$x=1 \Rightarrow u=0 \qquad x=e \Rightarrow u=1$

$$= \int_0^1 u\,du = \tfrac{1}{2}u^2\Big]_0^1 = \tfrac{1}{2}(1-0) = \tfrac{1}{2}$$

D 27. $x+y=3$, and $x>0, y>0$

Maximum value of x^3+12xy?

$y=3-x$

$f(x)= x^3+12x(3-x) = x^3+36x-12x^2$

$f'(x)= 3x^2+36-24x = 0$

$x^2-8x+12=0$

$(x-6)(x-2)=0$

$x=6$, $x=2$. But if $y=3-x$, $3-6<0$, so $x=2$

Then $f(2)= \cancel{8+3}\ 2^3+36\cdot 2-12\cdot 2^2$

$= 8+72-48 = 32$

E 28. $\frac{ds}{dt}=2$ cm/s Find $\frac{dA}{dt}$ when $V=27$

$V=27$ and $V=s^3 \Rightarrow s=3$

$A=6s^2$

$\frac{dA}{dt} = 12s\frac{ds}{dt} = 12\cdot 3\cdot 2 = 72$

A 29. $f'(x) = \tan^{-1}(x^3 - x)$ How many times is tangent parallel to $y = 2x$

$\tan^{-1}(x^3 - x) = 2$; ~~3 intersections on graph~~

no intersections.

C 30. I. $f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$ not defined at $x=0$; f is continuous

II. Cont. and ~~d~~ not diff at $x=0$

III $h(x) = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$ $h'(x) = \begin{cases} 2x, & x \geq 0 \\ -2x, & x < 0 \end{cases}$ both $= 0$ at $x = 0$

so cont AND diff. at $x=0$.

Thus I, II only

D 31. $v(t) = \sqrt{t} - \cos(e^t)$, $t \geq 0$ At $t=1$,

$v(1) = \sqrt{1} - \cos e > 0$ (0.001125)

~~$a(t) =$~~ $a(1) \doteq 0.502 > 0$ (nderiv.)

E 32 $f'(x) = \sin(2^x)$ $\forall x$.

Minimum when f' changes from $-$ to $+$, at 2.651

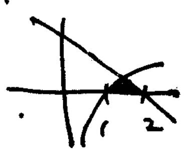
E 33. $\frac{1}{b-a}\int_a^b x^3\,dx = \frac{1}{b-a} \cdot \frac{1}{4}x^4\Big]_a^b = \frac{1}{4} \cdot \frac{b^4 - a^4}{b-a}$

A 34. $F'(x) = f(x)$. $\int_a^b x f(x^2)\,dx$ $\quad x=a \Rightarrow u = a^2$

$u = x^2$, $du = 2x\,dx$ $\quad x=b \Rightarrow u = b^2$

$$\frac{1}{2}\int_a^b 2x f(x^2)\,dx = \frac{1}{2}\int_{a^2}^{b^2} f(u)\,du = \frac{1}{2}F(u)\Big]_{a^2}^{b^2} = \frac{1}{2}\left(F(b^2) - F(a^2)\right)$$

B 35. $y = 2 - x$ $\quad y = \ln x$ $\quad$ x-axis



$$\int_1^a \ln x\,dx + \int_a^2 (2-x)\,dx$$

a = intersection of $y = 2-x$, $y = \ln x$

$a \approx 1.55715$

A 36. Average rate of change of g is $\frac{g(5)-g(0)}{5-0} = \frac{5-2}{5-0} = \frac{3}{5}$

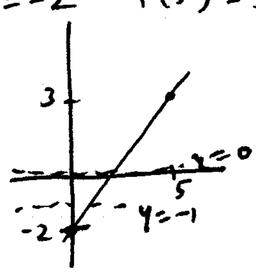
D 37. Growth in cm from $t=10$ to $t=20$ is $\int_{10}^{20} g(t)\,dt \approx 20+$

$60 + 20^{+} = 80^{+}$

C 38. Graph is g. Then g' is its slope. The most negative value of g' is at around $t=7$.

E 39. f is continuous on $[0,5]$, differentiable on $(0,5)$.

$f(0) = -2$ $\quad f(5) = 3$



slope from $(0,-2)$ to $(5,3)$ is $\frac{5}{5} = 1$

All true.

C 40 ~~t = temper~~ c = temp in °C t = time in minutes

$\frac{dV}{dc} = 6 \text{ in}^3/°c$ $V = \frac{4}{3}\pi r^3$, $V = 36\pi \text{ in}^3$.

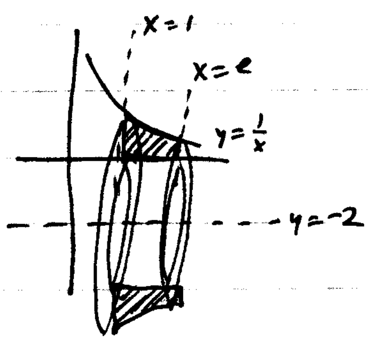
$\frac{dc}{dt} = 2\ °C/\text{min}$ Find $\frac{dr}{dt}$ in in/min

$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$ When $V = 36\pi$, $r = \sqrt[3]{\frac{3}{4}\cdot 36} = 3$ in.

$\frac{dV}{dt} = \frac{dV}{dc} \cdot \frac{dc}{dt} = 6 \cdot 2 = 12 \text{ in}^3/\text{min}$

$12 = 4\pi \cdot 3^2 \cdot \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{12}{4\pi \cdot 9} = \frac{12}{36\pi} = \frac{1}{3\pi}$

D 41.



$\pi \int_1^e (R^2 - r^2)\,dx$

$= \pi \int_1^e \left(\left(2 + \frac{1}{x}\right)^2 - 2^2\right) dx$

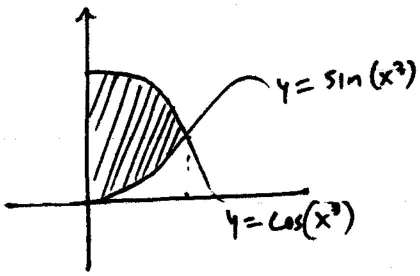
C 42. $g(x) = \int_0^{x/2} e^{-t^2}\,dt$ Tangent to g(x) at x = 1

Clearly y = 0.461 from choices

$g'(x) = e^{-(x/2)^2} \cdot \frac{1}{2}$

$g'(1) = e^{-(1/2)^2} \cdot \frac{1}{2} = \frac{1}{2} e^{-1/4} \approx 0.3894$

C 43.

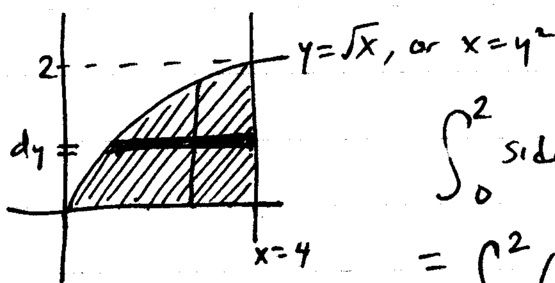


$\sin(x^3) = \cos(x^3)$

$x \approx 0.9226 \approx a$

$\int_0^a (\cos(x^3) - \sin(x^3))\,dx \approx 0.709$

E 44.



$\int_0^2 \text{side}^2\, dy = \cancel{\int_0^2 (y^2)^2\, dy}$

$= \int_0^2 (4 - y^2)^2\, dy$

B 45. leakage rate $= \ln(1+t) + \cos(t + \sqrt{t}) = f'(t)$

$f(0) = 375$

$f(60) = f(0) - \int_0^{60} f'(t)\,dt \approx \cancel{564.573}\ 185.427$

Practice Exam AB - 1 Free Response

1.

This question was replaced by a no-calculator question in the second edition of the book.

1. Tortoise velocity: $v(t) = \ln(1+t^2)$ $x(0)=0$

a) Avg acceleration $= \dfrac{v(10)-v(0)}{10-0} = \dfrac{\ln(101)-\ln(1)}{10-0} \approx \boxed{0.462 \text{ in/min}^2}$

Inst. acceleration $= a(t) = v'(t) = \dfrac{1}{1+t^2} \cdot 2t$

$a(10) = \dfrac{20}{1+10^2} = \dfrac{20}{101} \approx \boxed{0.198 \text{ in/min}^2}$

b) $v(t) \geq 0 \ \forall t$, so distance = displacement.

$\int_0^{10} v(t)\,dt \approx 29.09346\ldots \approx \boxed{29.093 \text{ inches}}$

c) Inst. velocity = avg velocity

$\ln(1+t^2) = \dfrac{29.093}{10}$ at $t \approx \boxed{4.165 \text{ min}}$

d) Hare has initial velocity of b at $t=9.5$, and acceleration of 2.
Hare's position at $t=9.5$ is $h(9.5)=0$, and position at $t=10$ is same as tortoise: $h(10) \approx 29.093$. Find b.

$\int_{9.5}^{10} h'(t)\,dt = 29.093$

$h'(t) = b + 2(t-9.5) = b + 2t - 19$

$\int_{9.5}^{10} (b+2t-19)\,dt = 29.093$

$\left[bt + t^2 - 19t\right]_{9.5}^{10} = (10b + 100 - 190) - (9.5b + 90.25 - 180.5)$

$= 0.5b + 0.25 \approx 29.093$

$\boxed{b \approx 57.687}$ in/min

2. $r(t) = e^{\sin(\frac{\pi}{4}t)}$ liters/h & initial amount $m(0) = 20$

At $t=8$, a pump removes at $p(t) = 1.5$ L/h.

a) When is methane increasing most rapidly on $t \in [0,8]$?

Maximum of $r(t)$ when $r'(t) = 0$ or at an endpoint.

$$r'(t) = e^{\sin(\frac{\pi}{4}t)} \cdot \cos(\tfrac{\pi}{4}t) \cdot \frac{\pi}{4} = 0$$

$$\frac{\pi}{4}t = \frac{\pi}{2}, \text{ or } t = 2$$

$$\frac{\pi}{4}t = \frac{3\pi}{2}, \text{ or } t = 6$$

$r(0) = e^0 = 1$

$r(2) = e^{\sin\frac{\pi}{2}} = e^1$

$r(6) = e^{\sin\frac{3\pi}{2}} = e^{-1} = \frac{1}{e}$

$r(8) = e^0 = 1$

Maximum is at $\boxed{t = 2 \text{ hours}}$.

b) Total methane at $t=8$:

$$m(8) = m(0) + \int_0^8 r(t)\,dt \approx \boxed{30.129 \text{ L}}$$

c) Average rate of methane accumulation over $t \in [0,24]$

$$= \frac{\text{total accumulation}}{24} = \frac{\int_0^8 r(t)\,dt + \int_8^{24} (r(t) - 1.5)\,dt}{24}$$

$$= \frac{6.38558}{24} \approx \boxed{0.266 \text{ L/h}}$$

d) Max amt. of methane can occur at an ~~±~~ endpoint or where the rate of change is 0 or undefined

Rate of change has a discontinuity at $t = 8$.

$r(t) = 0$ when $e^{\sin(\frac{\pi}{4}t)} - 1.5 = 0$ during $[8, 24]$, which happens several times. However, I only have to consider when $e^{\sin(\frac{\pi}{4}t)} - 1.5$ changes from + to −, at $t \approx 11.468$ and 19.468.

$m(t) =$ amount of methane at time t.

$m(0) = 20$

$m(8) \approx 30.129L$

$$m(11.468) \approx 30.129 + \int_8^{11.468} \left(e^{\sin(\frac{\pi}{4}t)} - 1.5\right) dt \approx 32.1736$$

$$m(19.468) \approx 30.129 + \int_8^{19.468} \left(e^{\sin(\frac{\pi}{4}t)} - 1.5\right) dt \approx 30.3021$$

Therefore abs. max. amount of methane is $\boxed{32.174\,L}$

3. $g(x) = \int_0^x f(t)\,dt \qquad g(3) = \frac{11}{5}$.

a) $g'(3) = f(3) = \boxed{2}$

$g''(3) = f'(3)$ = slope of pictured line = $\frac{2-0}{3-(-2)} = \boxed{\frac{2}{5}}$

b) The graph of g has a relative maximum when $g' = f$ changes from + to −, at $\boxed{x = -6 \text{ only}}$

c) The graph of g has a point of inflection when $g'' = f'$ changes signs, or when f changes direction.
This occurs at $\boxed{x = -3 \text{ and } x = 4}$.

d) $C(x) = \frac{1}{x-0} \int_0^x f(t)\,dt = \frac{g(x)}{x}$

$C(3) = \frac{1}{3} \int_0^3 f(t)\,dt = \frac{1}{3}\left(g(3) - g(0)\right) = \frac{1}{3}\left(\frac{11}{5} - 0\right) = \boxed{\frac{11}{15}}$

$\cancel{C'(3) = \frac{1}{3} f(3) = \frac{1}{3} \cdot 2 = \boxed{\frac{2}{3}}}$

$C'(x) = \frac{x g'(x) - g(x) \cdot 1}{x^2}$

$C'(3) = \frac{3 \cdot 2 - \frac{11}{5} \cdot 1}{3^2} = \frac{\frac{19}{5}}{9} = \boxed{\frac{19}{45}}$

4. a) $y' = -\sin\left(\frac{\pi x}{3}\right) \cdot \frac{\pi}{3}$

$\frac{x}{2} = \cos\left(\frac{\pi x}{3}\right)$ appears to be satisfied at $x=1$, so I will just check that:

$$\frac{1}{2} \overset{?}{=} \cos\left(\frac{\pi \cdot 1}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{2}.$$ Yep. P is $(1, \frac{1}{2})$

$$y'(1) = -\sin\left(\frac{\pi \cdot 1}{3}\right) \cdot \frac{\pi}{3} = -\frac{\sqrt{3}}{2} \cdot \frac{\pi}{3} = -\frac{\pi\sqrt{3}}{6}.$$

b) $$\int_0^1 \left(\cos\left(\frac{\pi x}{3}\right) - \frac{1}{2}x\right)dx = \left[\frac{3}{\pi}\sin\left(\frac{\pi x}{3}\right) - \frac{1}{4}x^2\right]_0^1$$

$$= \left(\frac{3}{\pi}\sin\frac{\pi}{3} - \frac{1}{4}\right) - \left(\frac{3}{\pi}\sin 0 - 0\right) = \frac{3}{\pi} \cdot \frac{\sqrt{3}}{2} - \frac{1}{4}$$

$$= \frac{3\sqrt{3}}{2\pi} - \frac{1}{4}.$$

c) S consists of a triangle on $[0, 1]$ and the area under that cosine curve from $x = 1$ to the x-intercept.

$$\cos\left(\frac{\pi x}{3}\right) = 0$$

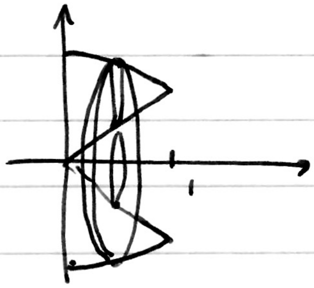
$$\frac{\pi x}{3} = \frac{\pi}{2} \Rightarrow 2\pi x = 3\pi \Rightarrow x = \frac{3}{2}.$$

$$\frac{1}{2} \cdot 1 \cdot \frac{1}{2} + \int_1^{3/2} \cos\left(\frac{\pi x}{3}\right)dx$$

$$= \frac{1}{4} + \left[\frac{3}{\pi}\sin\left(\frac{\pi}{3}x\right)\right]_1^{3/2} = \frac{1}{4} + \frac{3}{\pi}\left(\sin\frac{\pi}{2} - \sin\frac{\pi}{3}\right)$$

$$= \frac{1}{4} + \frac{3}{\pi}\left(1 - \frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{3}{\pi} - \frac{3\sqrt{3}}{2\pi}$$

d)



$$\pi \int_0^1 (R^2 - r^2)\, dx$$

$$= \pi \int_0^1 \left(\left(\cos\left(\frac{\pi x}{3}\right)\right)^2 - \left(\frac{x}{2}\right)^2 \right) dx$$

5. a) bases are each 2 units wide

$$\int_{-3}^{3} f(x)\,dx \approx 2\big(f(-2) + f(0) + f(2)\big) = 2(3 + -1 + -4) = 2(-2) = \boxed{-4}$$

b) Avg. r.o.c. of f over $[-3,0] =$ $\cancel{\frac{1}{0-(-3)}\int_{-3}^{0} f'}$

$$= \frac{1}{3}\big(f(0) - f(-3)\big) = \frac{1}{3}(-1-5) = -2$$

Avg. r.o.c. of f over $[0,3] =$

$$\frac{1}{3}\big(f(3) - f(0)\big) = \frac{1}{3}\big(-7-(-1)\big) = -2$$

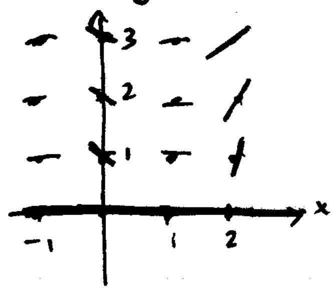
And these are equal.

c) Since f has a positive average rate of change on $[0,1]$, and a negative average r.o.c. on $[1,2]$, and f is known to be twice differentiable, and therefore continuous, $f'(c)$ must be equal to 0 ~~at~~ for some $c \in (0,2)$.

d) Yes. The average r.o.c. of f on $[-3,-2]$ is -2, and on $[-2,-1]$ is -1. This shows an increase in f'. The average r.o.c. of f on $[-1,0]$ is -3, indicating a decrease in f'. When f' changes from increasing to decreasing and is differentiable, f'' must be 0.

6. $\frac{dy}{dx} = \frac{x^2-1}{y}$ $\qquad f(2) = 1.$

a)



If $x = \pm 1$, $\frac{dy}{dx} = 0$

$(0,1) \Rightarrow m = -1$ $\qquad (2,1) \Rightarrow m = \frac{3}{1} = 3$

$(0,2) \Rightarrow m = \frac{-1}{2}$ $\qquad (2,2) \Rightarrow m = \frac{3}{2}$

$(0,3) \Rightarrow m = \frac{-1}{3}$ $\qquad (2,3) \Rightarrow m = \frac{3}{3}$

b) $f''(2)$:

$$\frac{d^2y}{dx^2} = \frac{y \cdot 2x - (x^2-1) \cdot \frac{dy}{dx}}{y^2}$$

At $(2,1)$, $\frac{dy}{dx} = 3$ (from above)

so $\left.\frac{d^2y}{dx^2}\right|_{(2,1)} = \frac{1 \cdot 4 - 3 \cdot 3}{1^2} = \boxed{-5}$

c) $\int y\, dy = \int (x^2-1)\, dx$

$\frac{1}{2}y^2 = \frac{1}{3}x^3 - x + C$

$\cancel{y^2 = \frac{2}{3}x^3 - x}$

$y^2 = \frac{2}{3}x^3 - 2x + C$

$y = \sqrt{\frac{2}{3}x^3 - 2x + C}$

$\boxed{y = \sqrt{\frac{2}{3}x^3 - 2x - \frac{1}{3}}}$

At $(2,1)$

$1 = \sqrt{\frac{2}{3} \cdot 2^3 - 2 \cdot 2 + C}$

$1 = \sqrt{\frac{16}{3} - 4 + C}$

$1 = \frac{4}{3} + C$

$C = -\frac{1}{3}$