

Limits

Analysis of graphs

Interplay between geometric and analytic information

Use of calculus to predict and explain observed local and global behavior of a function

Limits of functions (including one-sided limits)

An intuitive understanding of the limiting process

Calculating limits using algebra

Estimating limits from graphs or tables of data

Asymptotic and unbounded behavior

Understanding asymptotes in terms of graphical behavior

Describing asymptotic behavior in terms of limits involving infinity

Comparing relative magnitudes of functions and their rates of change (e. g., contrasting exponential growth, polynomial growth, and logarithmic growth)

Continuity as a property of functions

An intuitive understanding of continuity (close values of the domain lead to close values of the range)

Understanding continuity in terms of limits

Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)

Derivatives

Concept of the derivative

Derivative presented graphically, numerically, and analytically

Derivative interpreted as an instantaneous rate of change

Derivative defined as the limit of the difference quotient

Relationship between differentiability and continuity

Derivative at a point

Slope of a curve at a point, including vertical, nonexistent

Tangent line to a curve at a point; local linear approximation

Instantaneous rate of change as the limit of average rate of change

Approximate rate of change from graphs and tables of values

Derivative as a function

Corresponding characteristics of graphs of f and f'

Relationship between increasing and decreasing behavior of f and the sign of f'

The Mean Value Theorem and its geometric interpretation

Equations involving derivatives; translating words to symbols

Second derivatives

Corresponding characteristics of the graphs of f , f' , and f''

Relationship between the concavity of f and the sign of f''

Points of inflection as places where concavity changes

Applications of derivatives

- Analysis of curves, including monotonicity and concavity
- Optimization, both absolute (global) and relative (local) extrema
- Modeling rates of change, including related rates problems
- Use of implicit differentiation to find the derivative of an inverse
- Interpretation of derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations

Computation of derivatives

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trig
- Basic rules for derivatives of sums, products, and quotients
- Chain rule and implicit differentiation

Integrals

Interpretations and properties of definite integrals

- Definite integral as a limit of Riemann sums
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

- Basic properties of definite integrals (incl. additivity, linearity)

Applications of integrals

- Knowledge, techniques adapted to many applications
- Emphasis is on setting up approximating Riemann sum and representing its limit as a definite integral
- Specific applications include finding the area of a region, volume of a solid with known cross sections, average value of a function, distance traveled by a particle along a line, and accumulated change from a rate of change

Fundamental Theorem of Calculus

- Use of the Fundamental Theorem to evaluate definite integrals
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis

Techniques of antidifferentiation

- Antiderivatives following from derivatives of basic functions
- Antiderivatives by substitution of variables, incl. change of limits for definite integrals

Applications of antidifferentiation

- Finding specific antiderivatives using initial conditions, incl. applications to motion along a line
- Solving separable differential equations and using them in modeling. In particular, studying the equation $y' = ky$ and exponential growth

Numerical approximations to definite integrals

- Use of Riemann sums (using left, right, midpoint values) and trapezoidal sums to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values